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EFFECTS OF REDUCED ORDER MODELING ON THE CONTROL OF A LARGE SPACE STRUCTURE

by

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Thesis Advisor: Jeff B. Burl

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The motion of a large space structure, such as a space station, is described by a large number of coupled, second order differential equations. To effectively control this structure, a mathematical model is required. Both the mathematical model developed directly from the physics of the structure, and the simplified model developed with modal analysis are of extremely high dimension. A reduced order model is therefore required in order to design a control system for the structure.

A straightforward approach to the control problem is taken by using linear quadratic optimal control techniques to determine the reduced order control solution for the truncated modal model. The effects of reduced order modeling on the control of the space station will be evaluated by observing the response of the closed loop system to several disturbances.

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Effects of Reduced Order Modeling on the Control of a Large Space Structure

by

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ABSTRACT

The motion of a large space structure, such as a space station, is described by a large number of coupled, second order differential equations. To effectively control this structure, a mathematical model is required. Both the mathematical model developed directly from the physics of the structure, and the simplified model developed with modal analysis are of extremely high dimension. A reduced order model is therefore required in order to design a control system for the structure.

A straightforward approach to the control problem is taken by using linear quadratic optimal control techniques to determine the reduced order control solution for the truncated modal model. The effects of reduced order modeling on the control of the space station will be evaluated by observing the response of the closed loop system to several disturbances.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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ACKNOWLEDGEMENTS

I would like to express my appreciation to the McDonnell Douglas Astronautics Company of Huntington Beach, California, for providing the dynamic model of a preliminary space station configuration. Also, I would like to thank Professor Alan J. Laub of the University of California at Santa Barbara for the subroutine solution to the discrete-time Riccati equation. This was essential to the completion of this study. Finally, I would like to thank Professor J. B. Burl for his patience and assistance in the completion of this work.

This thesis is dedicated to [redacted] Without their support this work would not have been possible.

I. INTRODUCTION

A. BACKGROUND

A large space structure, such as a space station, presents numerous control problems to the engineer. One such problem involves the control of unwanted vibrations. These vibrations must be controlled to prevent the disruption of delicate scientific experiments and maintain the stability of the structure.

The lightweight materials used in the construction of the space station and its large size combine to form a flexible, lightly damped structure that will vibrate for a considerable length of time when disturbed. This thesis addresses the problem of actively controlling these vibrations. A representation of a dual keel space station, courtesy of McDonnell Douglas Astronautics, is presented in Figure 1.

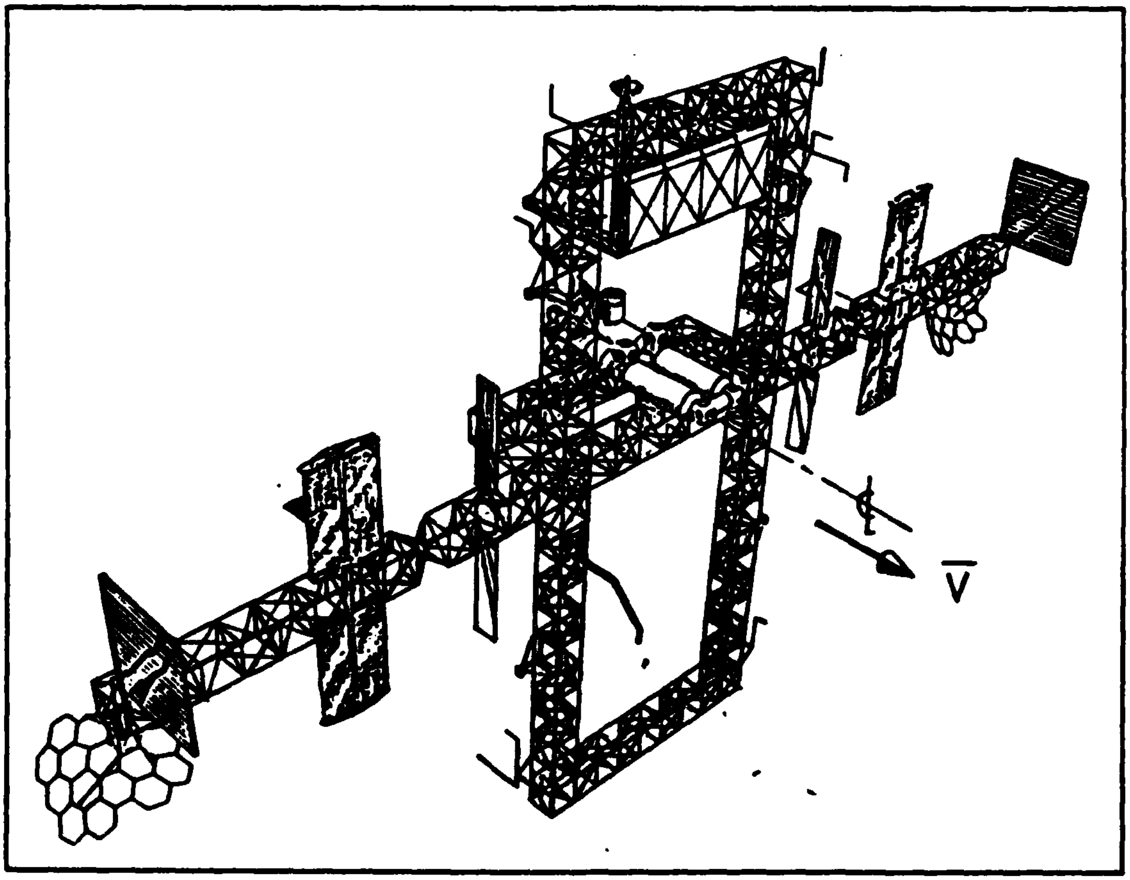

Figure 1. Representation of a Dual Keel Space Station.

B. PROBLEM STATEMENT

To solve the vibration control problem, several steps are required. First, a mathematical model must be developed that describes the behavior of the system over time. Modal analysis develops the modal model representing the system as a set of uncoupled, simultaneous, second order differential equations. The order of this model is high, making control design and implementation difficult. A reduced order model can be easily generated by truncating the modal equations.

Second, a control system must be designed using the mathematical model. The control system design is performed using optimal control techniques, that is, a control solution is found that will minimize a performance or cost function of the structure. In this case, the cost function will be the vibrational energy of the structure plus a term representing the control energy. This cost is then minimized through the development of an optimal feedback gain matrix.

Third, the controlled model must be simulated to determine the system response to a disturbance applied to various points on the structure. This simulation will utilize various optimal gain matrices based on a range of reduced order models. The effects of reduced order modeling on the control of the space station will be evaluated by observing the response of the closed loop system to several disturbances.

Finally, conclusions will be presented based on the results, and recommendations will be made for further areas of research.

C. ORGANIZATION

In Chapter II, the model of a space station will be developed. The modal model is developed and discretized to yield the discrete-time state equations used in the simulation. The data for this model was provided by the McDonnell Douglas Astronautics Company.

The desired performance function will be obtained in Chapter III, along with the optimal gain matrix necessary to minimize this function. Together, these provide the basis for the reduced order control solution.

Chapter IV presents the computer simulation of the model and the reduced order control solution. The results of reduced order control on the system will be evaluated.

Conclusions based on the simulation results are presented in Chapter V, as well as recommendations for areas of future investigation.

II. THE MATHEMATICAL MODEL

A. INTRODUCTION

A mathematical model describing the motion of the space station is required for effective control. Because of its size and construction, the space station can be considered to be a lightly damped, vibrating structure which, when disturbed by an external force, may vibrate for a considerable length of time.

The space station can be modeled as a finite number of discrete masses connected by springs and dashpots. This is a complex description of the system's motion and is quite difficult to work with. By expressing the equations of motion in terms of the structure's natural modes of vibration a simpler model will be obtained.

The natural modes of the system will form the basis for the continuous-time mathematical model, developed in section B, expressed in terms of an uncoupled set of second order differential equations. The discrete-time model will be derived in section C from the results of section B.

B. THE MODAL MODEL

The space station is a lightly damped structure consisting of numerous natural modes of vibration. The structure can be modeled as a system of discrete masses connected by springs and dashpots. In mechanical systems, the springs represent the stiffness factor and the dashpots represent the damping factor of the system. The displacement of the masses can be described by a second order matrix differential equation of motion:

$$\mathrm{M}\,\ddot{\mathrm{q}}(t) + \frac{\mathrm{d}}{\omega_f}\,\mathrm{K}\,\dot{\mathrm{q}}(t) + \mathrm{K}\,\mathrm{q}(t) = F(t) \tag{2.1}$$

where:

- q is the generalized coordinate vector
- M is the diagonal system mass matrix
- $\frac{\mathrm{d}}{\omega_f}\mathrm{K}$ is the structural damping term
- d is the damping constant
- ω_f is the frequency of oscillation of the system
- K is the symmetric system stiffness matrix

- F(t) is the system forcing function.

This equation represents a system of simultaneous, second order differential equations that are coupled through the K matrix.

Equation (2.1) can be decoupled by expressing q in terms of the natural modes of vibration. This is known as the process of modal analysis. The system is then represented by a set of independent differential equations that can be treated individually. Hurty and Rubinstein [Ref. 1] and Meirovitch [Ref. 2] outline the modal approach that forms the basis for the model developed in this section.

The first step in the model's development is the solution of the undamped, homogeneous form of Equation (2.1):

$$M\,\ddot{q}(t) + K\,q(t) = 0. \tag{2.2}$$

The solution to this equation can be found in any elementary differential equations textbook and can be written as:

$$q(t) = Ax\sin(\omega t + \Theta) \tag{2.3}$$

and

$$\ddot{q}(t) = -Ax\omega^2\sin(\omega t + \Theta). \tag{2.4}$$

Substituting these expressions into Equation (2.1) and solving:

$$\{K - \omega^2 M\}\,x = 0 \tag{2.5}$$

or

$$Kx = \omega^2 Mx. \tag{2.6}$$

Equation (2.5) is an eigenequation with n combinations of x and ω as solutions. Grouping the individual solutions:

$$X = [x_1\,x_2\,\ldots\,x_n]^T \tag{2.7}$$

$$\Omega^2 = diag[\omega_{o1}^2, \omega_{o2}^2, \ldots, \omega_n^2], \tag{2.8}$$

all solutions can be found by solving the matrix eigenvalue problem:

$$KX = \Omega^2 MX \tag{2.9}$$

where Ω^2 is the system natural frequency matrix and X is the system eigenvector or modal matrix. The individual elements of Ω are referred to as the natural frequencies, or eigenvalues of the system, and the columns of X are referred to as the natural mode shapes or the eigenvectors. Several properties of the eigenvector matrix are useful and can be developed by premultiplying Equation (2.9) by X^T:

$$X^T K X = \Omega^2 X^T M X. \tag{2.10}$$

The eigenvectors can be normalized:

$$X^T M X = I \tag{2.11}$$

where I is the identity matrix. From Equation (2.10), it follows:

$$X^T K X = \Omega^2 I = \Omega^2 \tag{2.12}$$

where Ω^2 is a diagonal matrix.

The equations of motion can be uncoupled through a linear transformation of the coordinate system [Ref. 3]:

$$q(t) = \sum_{i=1}^{n} x_i \eta_i(t) = X\eta(t) \tag{2.13}$$

where q(t) and η(t) are two different sets of generalized coordinate systems and

- X is the modal matrix
- n is the maximum number of degrees of freedom
- $\eta(t)$ is the transformed coordinate vector or the modal amplitude vector.

Applying the transformation to the system results in:

$$M X\ddot{\eta}(t) + \frac{d}{\omega_f} KX\dot{\eta}(t) + K X\eta(t) = F(t). \tag{2.14}$$

Multiplying both sides of Equation (2.14) by X^T:

$$X^T M X\ddot{\eta}(t) + \frac{d}{\omega_f} X^T KX\dot{\eta}(t) + X^T K X\eta(t) = X^T F(t). \tag{2.15}$$

Applying Equations (2.11) and (2.12), noting that for wideband excitation, $\omega_f \approx \omega_n$:

$$\ddot{\eta} + d\Omega\dot{\eta} + \Omega^2\eta = X^T F \tag{2.16}$$

where Ω has been established as a diagonal matrix. Equation (2.16) is the modal model, representing an uncoupled set of second order differential equations describing the motion of the structure in terms of its natural modes of vibration.

C. DISCRETE-TIME MODEL

The discrete-time, state space model is found by solving the continuous-time equations obtained in Section B. Solving the modal system of equations for the i^{th} solution determines the solution for all the individual second order differential equations of motion. The solution to this single equation will be used in the computer simulation.

The i^{th} equation of motion is expressed as:

$$\ddot{\eta}_i(t) + d\omega_{oi}\dot{\eta}_i(t) + \omega_{oi}^2\eta_i(t) = x_i^T F(t) \tag{2.17}$$

where:

- x_i^T is the transpose of the i^{th} mode shape vector
- F(t) is the torquing force applied at a point.

The homogeneous solution of this second order differential equation is:

$$\eta_i(t) = C_1\, e^{-d\frac{\omega_{oi}}{2}t} \cos(\mu t) + C_2\, e^{-d\frac{\omega_{oi}}{2}t} \sin(\mu t) \tag{2.18}$$

where

$$\mu = \frac{\sqrt{4\omega_{oi}^2 - d^2\omega_{oi}^2}}{2}. \tag{2.19}$$

By defining

$$\gamma = \frac{d\omega_{oi}}{2} \tag{2.20}$$

and

$$\mu = \omega_d = \sqrt{\omega_{oi}^2 - \gamma^2} \tag{2.21}$$

then substituting Equations (2.20) and (2.21) into Equation (2.18), the solution can be written:

$$\eta_i(t) = C_1\, e^{-\gamma t} \cos(\omega_d t) + C_2\, e^{-\gamma t} \sin(\omega_d t) \tag{2.22}$$

and

$$\dot{\eta}_f(t) = (C_2\omega_d - C_1\gamma)e^{-\gamma t}\cos(\omega_d t) - (C_1\omega_d + C_2\gamma)e^{-\gamma t}\sin(\omega_d t). \tag{2.23}$$

At $t = 0$, Equations (2.22) and (2.23) become:

$$\eta(0) = C_1 \tag{2.24}$$

and

$$\dot{\eta}(0) = C_2\omega_d - C_1\gamma. \tag{2.25}$$

Solving for C_1 and C_2:

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{\gamma}{\omega_d} & \frac{1}{\omega_d} \end{bmatrix} \begin{bmatrix} \eta(0) \\ \dot{\eta}(0) \end{bmatrix}. \tag{2.26}$$

The homogeneous solution to Equation (2.17) can then be written in terms of C_1 and C_2:

$$\begin{bmatrix} \eta(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} e^{-\gamma t}\cos(\omega_d t) & e^{-\gamma t}\sin(\omega_d t) \\ e^{-\gamma t}[\gamma\cos(\omega_d t) + \omega_d\sin(\omega_d t)] & e^{-\gamma t}[\omega_d\cos(\omega_d t) - \gamma\sin(\omega_d t)] \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}. \tag{2.27}$$

Substituting Equation (2.26) into Equation (2.27), this solution can be written in terms of the initial conditions:

$$\begin{bmatrix} \eta(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} e^{-\gamma t}\left[\cos(\omega_d t) + \frac{\gamma}{\omega_d}\sin(\omega_d t)\right] & \frac{1}{\omega_d}e^{-\gamma t}\sin(\omega_d t) \\ -\frac{\omega_o^2}{\omega_d}e^{-\gamma t}\sin(\omega_d t) & e^{-\gamma t}\left[\cos(\omega_d t) - \frac{\gamma}{\omega_d}\sin(\omega_d t)\right] \end{bmatrix} \begin{bmatrix} \eta(0) \\ \dot{\eta}(0) \end{bmatrix}. \tag{2.28}$$

Letting

$$Z_f(t) = \begin{bmatrix} \eta_f(t) \\ \dot{\eta}_f(t) \end{bmatrix} \tag{2.29}$$

and

$$\Phi_i = \begin{bmatrix} e^{-\gamma t}\left[\cos(\omega_d t) + \frac{\gamma}{\omega_d}\sin(\omega_d t)\right] & \frac{1}{\omega_d} e^{-\gamma t}\sin(\omega_d t) \\ -\frac{\omega_o^2}{\omega_d} e^{-\gamma t}\sin(\omega_d t) & e^{-\gamma t}\left[\cos(\omega_d t) - \frac{\gamma}{\omega_d}\sin(\omega_d t)\right] \end{bmatrix}, \tag{2.30}$$

the solution can be written:

$$\dot{Z}_i(t) = \Phi_i(t) Z_i(t) \tag{2.31}$$

where Φ_i is the state transition matrix of the i^{th} mode. The nonhomogeneous solution is obtained from:

$$\dot{Z}_i(t) = \Phi_i(t) Z_i(t) + \Gamma_i x_i^T F(t) \tag{2.32}$$

where the discrete-time input matrix is given by:

$$\Gamma_i = \int_0^T \Phi_i(\tau) B \partial\tau, \tag{2.33}$$

$B = [0 \;\; 1]^T$ is the input matrix for the continuous-time system, and T is the sampling time [Ref. 4: p. 59]. Solving Equation (2.33) yields:

$$\Gamma_i = \begin{bmatrix} \frac{1}{\omega_o^2}\left[1 - e^{-\gamma T}\cos(\omega_d T) - \frac{\gamma}{\omega_d} e^{-\gamma T}\sin(\omega_d T)\right] \\ \frac{1}{\omega_d} e^{-\gamma T}\sin(\omega_d T) \end{bmatrix}. \tag{2.34}$$

The discrete-time state equation for the i^{th} equation of motion can now be written:

$$Z_i(kT + 1) = \Phi_i(T)\, Z_i(kT) + \Gamma_i(T) x_i^T F(kT) \tag{2.35}$$

where Φ_i and Γ_i are evaluated at $t = T$. Summarizing the terms:

- Z_i is a vector of the i^{th} modal amplitude and the i^{th} modal velocity
- Φ_i is the i^{th} state transition matrix
- Γ_i is the i^{th} input vector
- x_i^T is the transpose of the i^{th} mode shape vector
- F is the control torque force vector applied at a point
- T is the sampling time

- k is the time index.

Equation (2.35) can be expanded to include a disturbance input:

$$Z_i(kT+1) = \Phi_i(T)\,Z_i(kT) + \Gamma_i(T)x_i^T[F(kT) + w(kT)] \tag{2.36}$$

where w(kT) is the disturbance input. Equation (2.36) is the discrete-time mathematical modal model describing the motion of the space structure in terms of its natural modes of vibration. The computer simulation of Chapter IV will solve the equations of motion by iterating this discrete-time model.

III. THE CONTROL SOLUTION

A. INTRODUCTION

The mathematical model derived in Chapter II represents a system with a control input and a random disturbance input. The problem that must be solved now is the development of the input that will effectively control the structure. The plant order is sufficiently large to preclude all but an optimal approach to the control problem [Ref. 4: p. 337]. Optimal control techniques attempt to find a control law which forces the system to follow a path that minimizes a given performance measure. [Ref. 5: p. 11]

Control system optimization involves the selection of a performance measure that describes a given characteristic, or property, of the system. Minimization of the performance measure is achieved by calculating the control gains and applying them to the system through state feedback. A suitable performance function based on the vibrational energy of the structure will be developed in Section B. This performance function, also referred to as the cost function, will provide a means of comparing various control models. The solution to the discrete-time Riccati equation, providing the necessary feedback gains required for an optimal control scheme, will be presented in Section C. A control system for the space station is then generated by applying the optimal control solution developed in this chapter to either the full order or reduced order model.

B. SYSTEM PERFORMANCE EVALUATION

The system mathematical model has been established. An expression must now be derived that will quantify control system performance and allow for a comparisons between competing systems. Kirk states:

> In selecting a performance measure the designer attempts to define a mathematical expression which when minimized indicates that the system is performing in the most desirable manner. Thus, choosing a performance measure is a translation of the systems physical requirements into mathematical terms [Ref. 5: p. 34].

Structural vibration is a physical property of the space station that disrupts the environment within the system. Vibrational energy in the structure can be due to any number of disturbances, for example:

- space shuttle docking
- rotating machinery
- positioning jet reaction

- movement of crew members
- rotation of solar panels.

A reasonable performance function is based on the vibrational energy of the structure. This performance function results in the control forces needed to minimize this vibration.

The performance measure is placed in a precise mathematical framework:

$$J = E[T.E.] + E[u^T R u] \tag{3.1}$$

where

- $E[\bullet]$ is the expected value of $\bullet$
- T.E. is the total vibrational energy
- $u^T R u$ limits the magnitude of the control force.

The total energy consists of the potential and kinetic energies of the structure at any point in time. This can be expressed in matrix form:

$$T.E.(t) = P.E.(t) + K.E.(t) = \frac{1}{2}\left[q^T(t)Kq(t) + \dot{q}^T(t)M\dot{q}(t)\right]. \tag{3.2}$$

Applying the coordinate transformation of Chapter II, the energy equations are written in terms of the modal amplitudes and modal velocities as:

$$P.E.(k) = \frac{1}{2}\sum_{i=1}^{n}\omega_{oi}^2\eta_i^2(k) \tag{3.3}$$

and

$$K.E.(k) = \frac{1}{2}\sum_{i=1}^{n}\dot{\eta}_i^2(k). \tag{3.4}$$

Note that the energy terms are summed over the n modes of the system. Defining the state weighting matrix Q_i as:

$$Q_i = \begin{bmatrix} \omega_{oi}^2 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.5}$$

permits the total energy to be written in terms of the modal state vectors as:

$$\mathrm{T.E.}(k) = \frac{1}{2}\sum_{i=1}^{n}\left[\eta_i(k)\;\dot{\eta}_i(k)\right] Q_i \begin{bmatrix} \eta_i(k) \\ \dot{\eta}_i(k) \end{bmatrix}, \tag{3.6}$$

and applying Equation (2.29) results in:

$$\mathrm{T.E.}(k) = \frac{1}{2}\sum_{i=1}^{n} Z_i^T(k) Q_i Z_i(k). \tag{3.7}$$

The cost function of Equation (3.1) can now be written:

$$J = E\left[\frac{1}{2}\sum_{i=1}^{n} Z_i^T(k) Q Z_i(k) + u^T(k) R u(k)\right] \tag{3.8}$$

where J is a quadratic performance measure. A great deal of theory is available concerning the solution of quadratic optimal control problems. [Ref. 6: p. 84]

The system performance will be found by computer simulation and comparisons of various reduced order models will be made. Evaluation of the expected values by monte carlo methods is tedious. An alternative approach is to compute the expected values from the impulse response:

$$J = \sum_{k=0}^{\infty}\left[\frac{1}{2}\sum_{i=1}^{n}\left[h_{\eta_i}^T(k)\,\omega_{oi}^2\,h_{\eta_i}(k) + h_{\dot{\eta}_i}^T(k)\,h_{\dot{\eta}_i}(k)\right] + h_u^T(k) R h_u(k)\right] \tag{3.9}$$

where

- $h_\eta(k)$ is the impulse response of η
- $h_{\dot{\eta}}(k)$ is the impulse response of $\dot{\eta}$
- $h_u(k)$ is the impulse response of u.

This equation can be easily evaluated by computer simulation. [Ref. 6: p. 85]

C. RICCATI SOLUTION

The solution to the quadratic optimal control problem is well known and extensively documented [Ref. 4: p. 338]. The optimal gain matrix must be chosen so Equation (3.8) will be minimized when the feedback loop is closed in Equation (2.36). The optimal control is state feedback:

$$F(k) = u(k) = LZ(k) \tag{3.10}$$

where the optimal gain matrix, L, is computed:

$$L = -(R + (\Gamma X_c^T)^T S \Gamma X_c^T)^{-1} (\Gamma X_c^T)^T S \Phi \tag{3.11}$$

and

- S is the steady-state matrix solution of the Riccati equation
- X_c^T is the mode shape matrix of the control node.

The product of the gain matrix and the time varying state matrix, Z, defines the control torque vector, u(k).

The solution matrix, S, is found by solving the discrete-time Riccati equation[1]:

$$S = \Phi^T S \Phi - \Phi^T S \Gamma X_c^T (R + (\Gamma X_c^T)^T S \Gamma X_c^T)^{-1} (\Gamma X_c^T)^T S \Phi + Q. \tag{3.12}$$

The solution to the Riccati equation is achieved by a variation of the Hamiltonian-Eigenvector approach. [Ref. 7,8]

The physical control of the structure is achieved by a system of control moment gyros or torquers. The mode shape vector X_i comprises the six degrees of freedom of the i^{th} mode: three degrees of modal deflection and three degrees of modal slope. For these point torquers, only the modal slopes at the torquer location are of concern in the development of the control input vector. The input matrix can then be simplified:

$$B = \Gamma X_c^T = \Gamma x'_c. \tag{3.13}$$

where x'_c is the row matrix of the modal slopes at the control moment gyro location. This simplification can also be applied to the point input disturbance where:

$$BN = \Gamma x'_n \tag{3.14}$$

and

- BN is the noise input matrix
- x'_n is the modal slope matrix associated with the disturbance node.

The discrete-time state of Equation (2.36) can now be written as:

---

1 The subroutine to solve the discrete-time Riccati equation was provided by Prof. Alan J. Laub, University of California, Santa Barbara.

$$Z(k+1) = \Phi Z(k) + B\,u(k) + BN\,w(k) \tag{3.15}$$

where w(k) is the applied disturbance input.

The space station has now been completely defined. It exists as a discrete-time mathematical model, consisting of a set of i uncoupled simultaneous differential equations of motion, expressed in terms of the natural modes of vibration. The control system is defined by the performance measure and either the full order or reduced order model. The vibrational energy of the structure is the measure of performance by which the reduced order control system will be judged. Input to the discrete-time Riccati equation consists of the state transition and input matrices of the model as well as the state weighting matrix and the modal slopes. The optimal control torques are based on the solution of this equation. The final product is Equation (3.15) which forms the framework for the simulation of Chapter IV.

IV. SIMULATION AND RESULTS

A. INTRODUCTION

The objective of the simulation is to determine the system response to disturbances applied at various points on the structure. This is accomplished by iterating the discrete-time model developed in Chapter II.

The process outlined above is covered in three sections. The specifics of the data used to model the space structure are discussed in Section B. An overview of the simulation program is given in Section C, and the specific disturbance used to excite the structure is introduced. The results of the simulation for various control conditions and disturbance locations are presented and discussed in Section D.

B. MODEL DATA

The dynamic model used in the simulation is for a preliminary space station configuration; the phase II dual keel structure[2]. The model consists of the first 100 natural modes, i.e., the natural frequencies for the first 100 modes and a matrix of 100 mode shapes for 114 nodes with six degrees of freedom at each node. The natural frequencies of the structure are shown in Table 1. The first six natural frequencies correspond to rigid body modes with the first bending mode beginning at number seven. Development of the reduced order control will be based on these bending modes. The modal amplitude data was normalized for a modal mass of 1 $\frac{\text{lb-s}^2}{\text{in}}$, specifying that the units for the simulation be given in the English system.

The system of 114 nodes is quite large, and it would be difficult to observe the response due to disturbances at each node. Therefore, three nodes of particular interest are considered in the simulation. They are the shuttle docking point at node 23, the alpha-joint[3] at node 55, and the location of the control moment gyros at node 69. The relative locations of these nodes on the structure is depicted in Figure 2 on page 17. These particular nodes are singled out for consideration because they are the point locations of either the applied control forces (node 69) or the applied disturbance force

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[2] The dynamic model for a preliminary space station configuration is provided courtesy of McDonnell Douglas Astronautics Company, 5301 Bolsa Avenue, Huntington Beach, CA 92647.

[3] The alpha-joint is the physical connection between the fixed structure and the rotating solar panels.

(nodes 23 or 55). The modal slopes associated with these particular nodes are listed in Appendix B.

Table 1. NATURAL FREQUENCIES OF THE SYSTEM.

MODE	NATURAL FREQUENCIES (RAD/SEC)			
1-- 4	0.000000	0.000000	0.000000	0.000000
5-- 8	0.000000	0.000000	0.568990	0.589359
9-- 12	0.615521	0.617321	0.629436	0.636641
13-- 16	0.638447	0.644367	0.657092	0.666360
17-- 20	0.782755	0.872754	1.003968	1.176393
21-- 24	1.377384	1.386371	1.432527	1.548961
25-- 28	1.839599	1.938523	2.227364	2.502074
29-- 32	2.893535	3.237640	3.290682	3.374448
33-- 36	3.861861	4.025045	4.168060	4.304161
37-- 40	4.551648	4.912770	5.023232	5.760018
41-- 44	6.342151	6.616597	6.728905	7.930507
45-- 48	8.679708	9.228074	9.478824	9.955171
49-- 52	10.273631	10.868124	11.217507	11.517843
53-- 56	11.771471	12.086158	13.359929	13.580529
57-- 60	14.019046	15.184227	15.480412	15.936637
61-- 64	17.897186	18.626617	20.483673	21.206009
65-- 68	23.067856	24.457626	25.562012	26.497147
69-- 72	27.506271	29.056931	30.375656	31.485458
73-- 76	31.687027	34.245392	35.777115	37.533356
77-- 80	39.633835	39.815964	40.360901	40.688721
81-- 84	42.038925	42.276260	44.139099	46.143112
85-- 88	46.860184	48.904663	49.485962	51.044510
89-- 92	51.501099	52.406784	55.534714	59.416992
93-- 96	62.643478	62.981720	65.679138	67.077789
97--100	67.632050	70.915939	75.196777	78.000229

C. SIMULATION PROGRAM

The Space Structure Simulation Program, listed in Appendix A, is written to simulate the space structure using the discrete mathematical model developed in Chapter II and the applied control developed in Chapter III. It provides the iterative solutions to Equation (3.15) and computes the cost function, Equation (3.8), using the equivalent form, Equation (3.9).

The program is structured in block format with each block providing the major input to the next. The major blocks, or sections, are:

- screen interaction
- establishment of the required matrices

- solution of the discrete-time Riccati equation and calculation of the optimal control gain matrix
- time iteration.

A discussion of these blocks will follow.

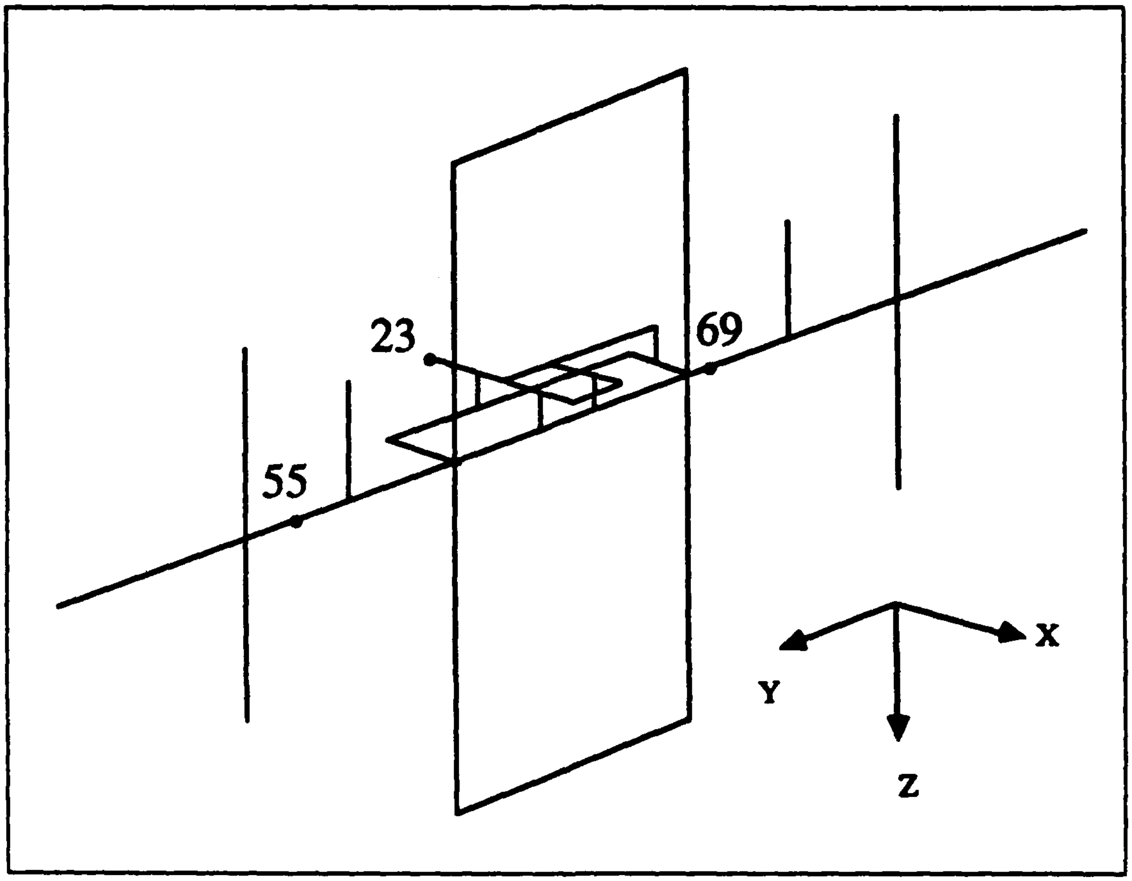

Figure 2. Node Location Diagram.

The simulation program begins with an interactive phase. This capability allows the user to select the values of certain parameters that are not fixed by either the program or the input data. The parameters that may be varied are:

- maximum size of the modal model
- number of control modes to be used, i.e., the size of the reduced order model
- node location of the applied disturbance
- axial direction of the applied disturbance
- sampling time

- damping factor
- value of the diagonal elements, r, of the control weighting matrix, R.

The choice of values for these items will fully specify the entire simulation. Several of these values were kept constant in generating the results presented in this thesis. They are:

- initial value of r: $1\text{x}10^{-12}$
- sampling time: $1\text{x}10^{-2}$
- damping factor: $1\text{x}10^{-3}$.

The value of r was picked based on trial and error, and represents a compromise between the control cost and the vibrational energy of the structure. The sampling time was based on the period of the highest frequency of the system. The chosen value represents a sampling frequency that is approximately ten times faster than the highest natural frequency, resulting in a minimal amount of aliasing. The damping factor was chosen to yield a lightly damped structure. This choice is based on previous space station studies that have used values in the range of 0.0001 to 0.005 [Ref. 9,10].

The program proceeds to construct the matrices of Equation (3.15) based on the choice of parameters, the natural frequencies of Table 1 on page 16, and the modal slopes at the control node and the disturbance node. The state weighting matrix, Q, is formed in accordance with Equation (3.5), and the control weighting matrix, R, is determined by the users selection of an appropriate value r where:

$$R = r\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4.1}$$

The next section of the program has two functions. The first is to obtain the discrete-time Riccati solution [Ref. 7,8] as given by Equation (3.12). This is accomplished by applying the subroutine, RICDSD, to the appropriate matrices of the previous paragraph. The second function is to apply the solution to obtain the optimal control gain matrix. A sample of this portion of the program is shown in Figure 3 on page 19. This sample includes the Riccati solution matrix, the closed loop eigenvalues of the reduced order model, and the control gain matrix. This information is provided as an output file for any sized reduced order model. At this point, all constant value matrices have been established.

```
STARTING MODE NUMBER:        7
NUMBER OF MODES SCANNED:     2
LAST CONTROLLED MODE:        8
NOISE INPUT NODE:           55
INITIAL R VALUE:             0.1000E-11
SAMPLING TIME:               0.1000E-01
DAMPING FACTOR:              0.1000E-02
OBSERVATION TIME:          120.0 MINUTES
SIZE OF MODAL MODEL:       100 MODES


THE RICCATI SOLUTION IS:

   0.593595E+02   0.402677E+01   -.207784E-01   -.349359E-01
   0.402677E+01   0.817013E+01   -.359488E-01   -.728645E-01
   -.207784E-01   -.359488E-01   0.619850E+02   0.485536E+01
   -.349359E-01   -.728645E-01   0.485536E+01   0.957929E+01


THE CLOSED LOOP EIGENVALUES ARE:

             Real                         Imaginary

   0.873338017523096283       0.000000000000000000E+00
   0.891377679206653312       0.000000000000000000E+00
   0.994092984040947231       0.000000000000000000E+00
   0.994306266098994654       0.000000000000000000E+00


THE CONTROL GAIN MATRIX L IS:

   -.502613E+06   -.962951E+06   -.647580E+05   -.121127E+06
   0.321279E+05   0.617015E+05   -.109143E+06   -.205384E+06
   0.571026E+05   0.110078E+06   -.509876E+06   -.959351E+06
```

Figure 3. Sample Output for a Two Mode Model of Reduced Order Control.

The time iterating portion of the simulation comprises the final major block of the simulation. It consists of a number of sub-sections designed to accomplish a specific sequence of tasks. The first sub-section computes the control torques based on the control or feedback gain matrix and the time varying state vector. The control torques form a $3x1$ vector and are obtained by the process of Equation (3.10). For the initial conditions at $k = 0$, the control is:

$$u(0) = L\,Z(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{4.2}$$

A new torque vector will be computed for each increment of k.

The disturbance input used in the simulation is a unit impulse. The impulse imparts a disturbance, at time $k = 0$, to a selected point (node 23, 55, or 69) on the structure, and the system response is then observed for the desired period of time. The disturbance may be applied in the x, y, or z direction, depending on the users choice during screen interaction. Provisions are also made within the program to obtain an uncontrolled impulse response.

The system cost function comprises the next sub-section of this block. The cost function is computed using Equation (3.9) with the total energy (T.E.) of the structure summed over all i modes for each k^{th} increment of time. This energy is combined with the control cost, u^TRu, to yield the cost function, J. In addition to the system cost, this section computes the total energy for each mode. The graphical analysis in Section D is based on the tabulated results of this sub-section.

The final sub-section of the block is the update of the discrete state equation. Equation (3.15) is updated over i modes for each k^{th} time increment and the new state vector Z results. For time $k = 0$, this equation can be written:

$$Z(1) = \Phi Z(0) + B u(0) + BN \delta(0) \tag{4.3}$$

where $\delta(0)$ is the applied unit impulse. When $k > 0$, Equation (4.3) reduces to:

$$Z(k+1) = \Phi Z(k) + B u(k). \tag{4.4}$$

The program loops back to the beginning of the time-varying block when the update of the state vector is complete. A new torque vector is computed, and the process described above continues until the response of the system has been found over the desired time period.

When the simulation is complete, the user is prompted to choose between running again with a new control weighting matrix, R, or termination of the simulation. The choice of a new weighting matrix reinitiates the simulation process with all parameters remaining the same except for the matrix R.

Output data can be tailored to fit user requirements. The program is written to provide the total system energy, the control cost, and the total energy per mode for a control based on a given reduced order model.

D. RESULTS

The response of the system to a disturbing force is provided by the simulation of Section C. The simulation is used to compute the systems response to a disturbance applied at either the space shuttle docking point, node 23, or the alpha-joint, node 55. These points frequently experience large disturbance inputs.

The simulation provides the energy of the vibrating structure in two formats. First, the total energy of the structure, as defined by Equation (3.8), is obtained. This provides a single value for a given condition of control. Second, the total energy of each mode is obtained. Essentially, this breaks down the single total energy value into 100 components, each representing the total energy of an individual mode. Tabulating the energy per mode will permit a more detailed look at the system response and the transfer of energy between the control system and the structure. Figure 4 on page 22 shows the full order system response due to a disturbance applied at each of the nodes of interest. Observe that the cost of control in terms of energy increases as the size of the reduced order control increases, but then begins to decrease. The important point to note is the size of the reduced order control required to bring the cost back to, and below, the uncontrolled level.

A major drawback arose during the simulation process. To obtain a single data point of Figure 4 on page 22 it required 40 minutes of computer CPU time to simulate the system. If the size of the model could be reduced and a similar response shape obtained, then the system could be simulated more efficiently.

Because high frequencies damp out quicker, the system was truncated at 56 modes; reasoning that the higher modes will die out quickly and contribute little to the cost. This size model would allow for up to 50 modes of control. Combined with the six rigid body modes, the system could then be simulated for full control, something that could not be achieved with a full order model. The model was truncated, and the results are shown in Figure 5 on page 23. The response very closely matches the shape of Figure 4 on page 22 for less than 30 modes of control. Above 30 modes, the comparison is less exact, but is still decreasing in energy. The significance of the truncated model is that it requires one-fifth of the CPU time (eight minutes vs. 40 minutes) to simulate the system for a two hour period. A two hour simulation period is used because this allows observation of the system for two time constants of the lowest natural frequency.

To use the truncated model, the energy content of the higher modes must be negligible. A comparison of the energy plots of both models indicate that the total energy

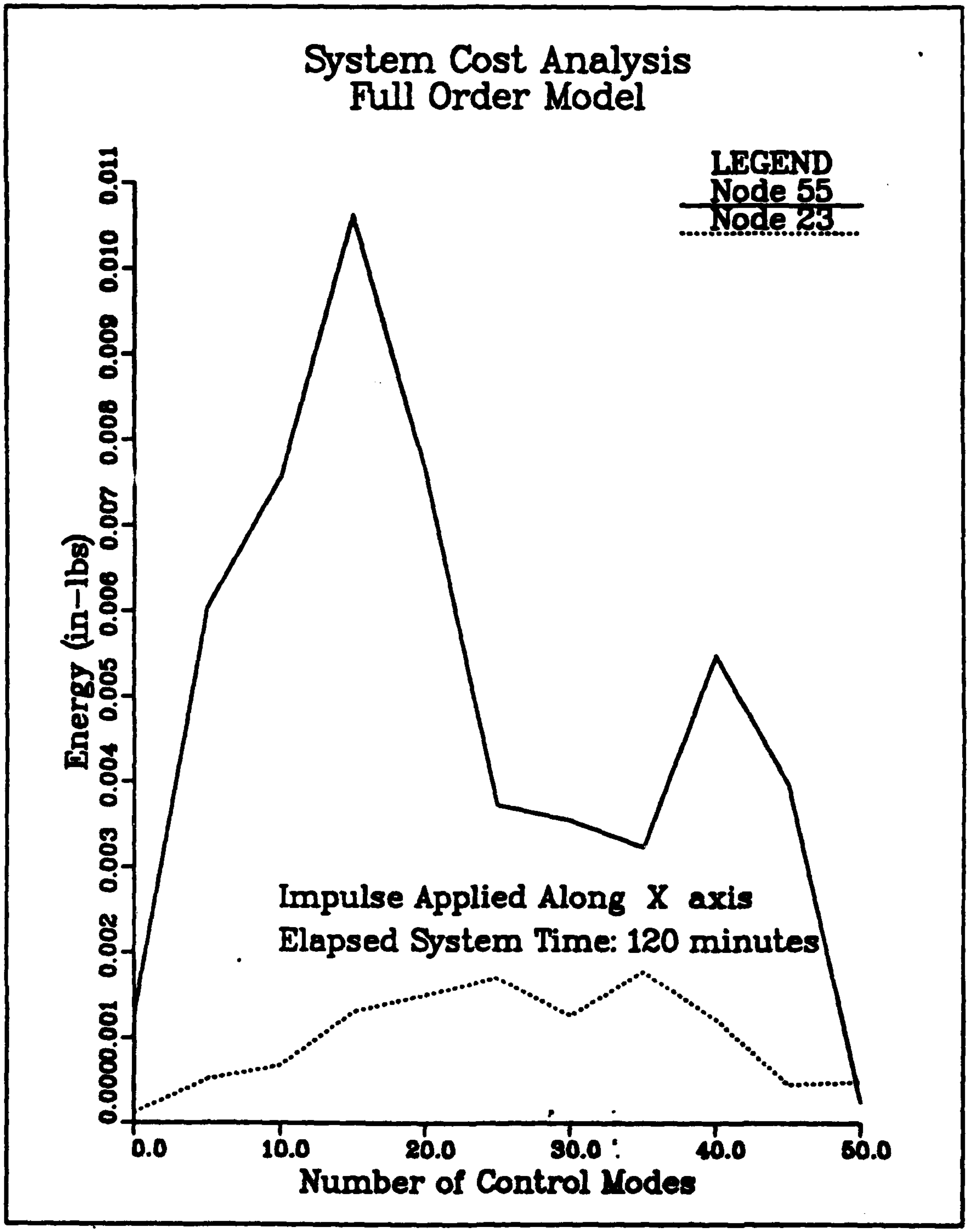

Figure 4. System Energy of a Full Order Model with Reduced Order Control.

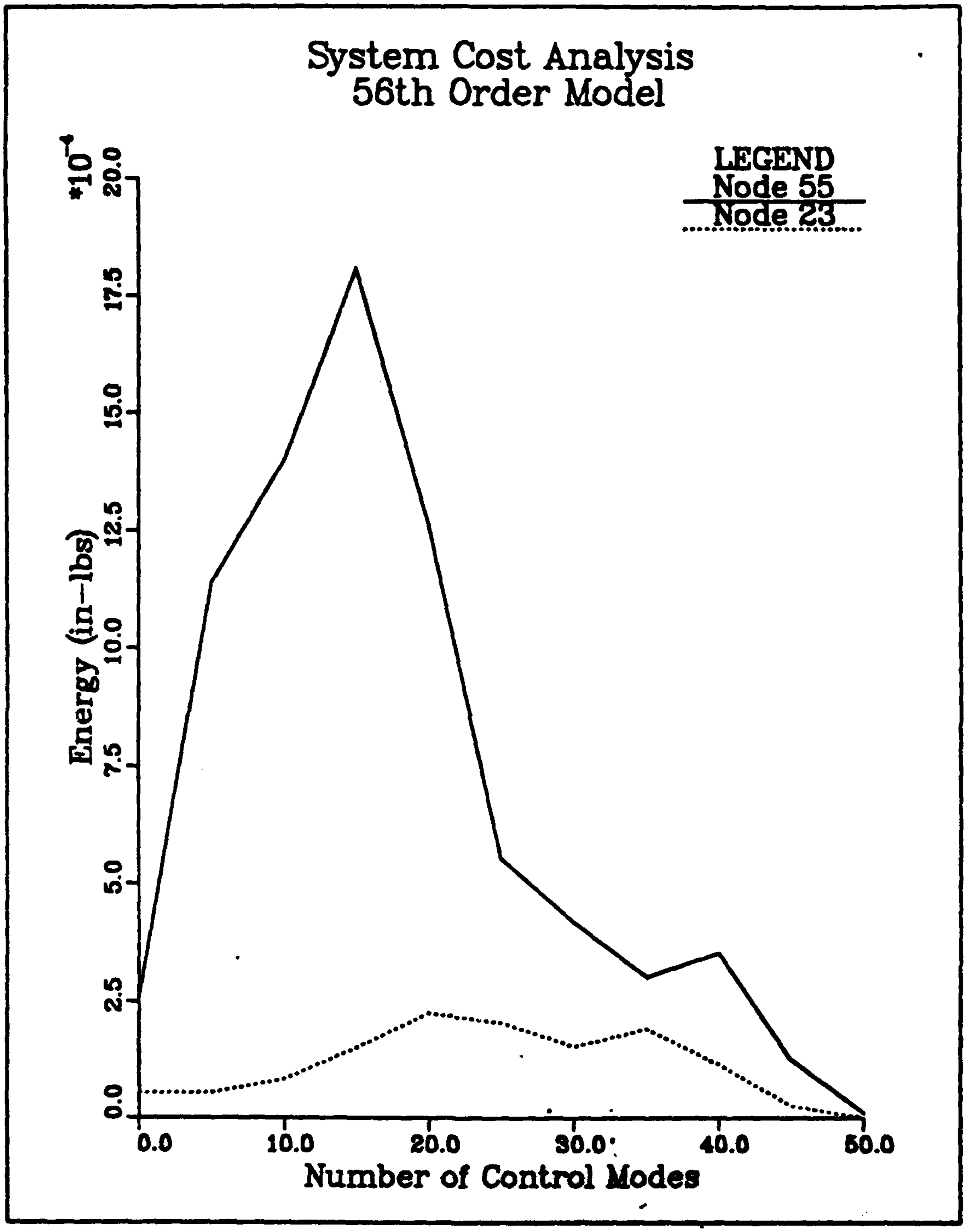

Figure 5. System Energy of a Reduced Order Model with Reduced Order Control.

of both models is nearly zero at 50 modes of control. The full order system still has 44 modes contributing to the energy cost while the reduced order model has none. The conclusion is reached that a good approximation of system response can be made by reducing the order of the model. The savings of computer costs is considerable when a number of control conditions are simulated.

A limitation of the total energy depiction of Figure 4 on page 22 and Figure 5 on page 23 is that an insight to the behavior of each mode is not provided. Figure 6 on page 25 thru Figure 15 on page 34 provide a graphical indication of mode response to an impulse applied at node 55. Each figure is a depiction of the energy in each mode for a given size of reduced order control compared to the uncontrolled system.

These ten figures show that the response of a directly controlled mode is essentially zero. They also show that as the number of controlled modes changes, there are certain groupings of modes that significantly increase their contribution to the cost over the uncontrolled system. These trouble modes appear in the areas of modes 30 to 32, 50 to 52, and 54 to 56. All other modes show little response to a change in the number of controlled modes.

A look at the system response to a disturbance applied at node 23 is shown in Figure 16 on page 35 thru Figure 25 on page 44. Note that the trouble modes of a node 23 response are the same as for a node 55 response. Although the shape of the uncontrolled and the controlled responses differ between the nodes, the offending modes remain the same.

The results indicate that there are certain modes, or groups of modes, that are troublesome. The same modes cause problems irregardless of whether the disturbance is applied to node 23 or node 55. The trouble modes must therefore be modes with large coupling to the control torquers. Two things influence this coupling, the natural frequency of the mode and the modal slope at the torquer location. As the natural frequency goes up the damping goes up proportionally but the spread between frequencies is small so damping is comparable for all modes. The modal slopes for the trouble modes (see Appendix B) indicate larger magnitude than those of the surrounding modes. The effect of these slopes is to increase the influence factor of the torquers. This can lead to a large excitation of the modes by the control system. If this is the case, it may be possible to identify possible trouble modes by comparing natural frequency spread with control node slope magnitude. A combination of a small frequency spread and a large slope magnitude could tag potential problem modes before simulation.

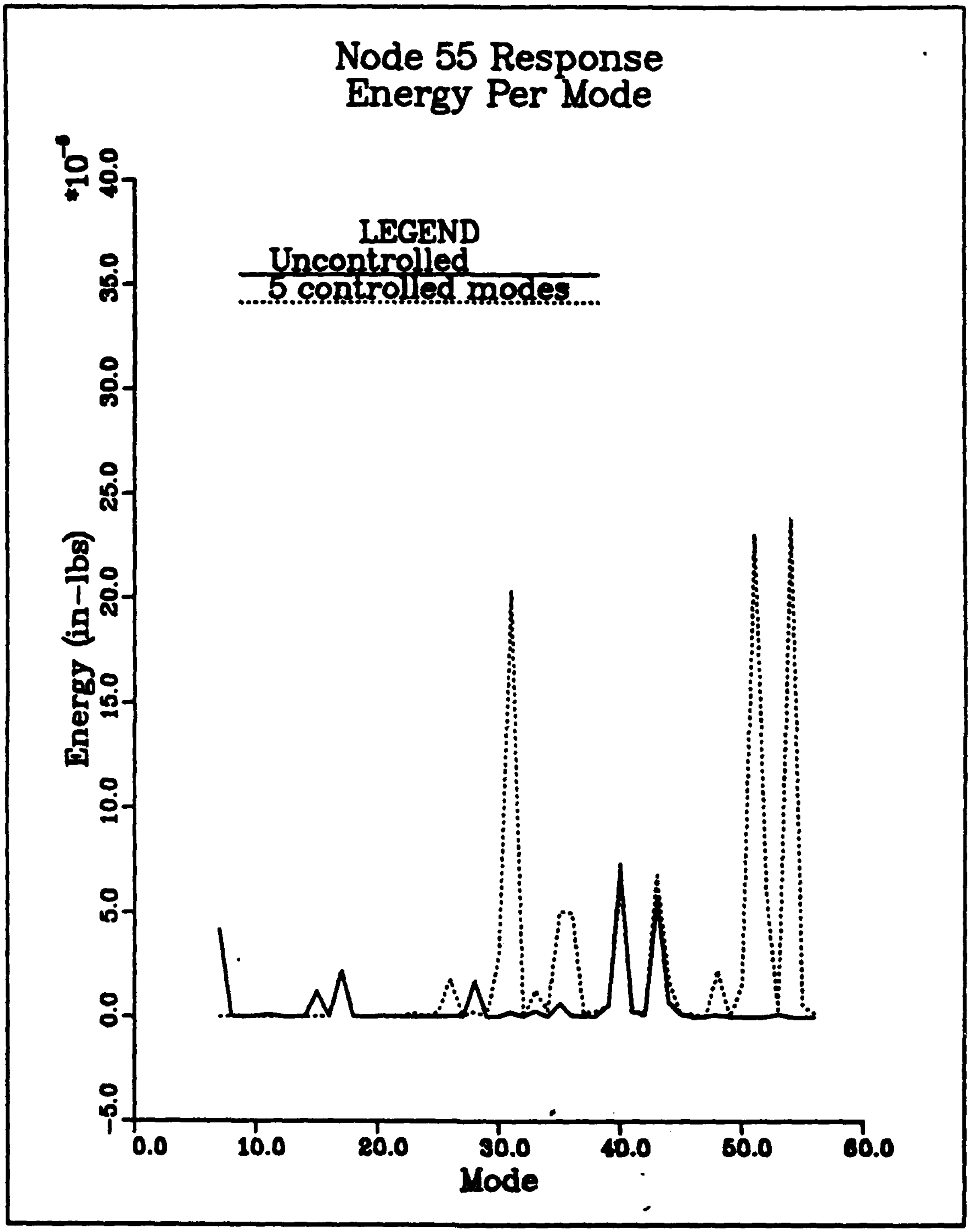

Figure 6. Node 55 Response with 5 Controlled Modes.

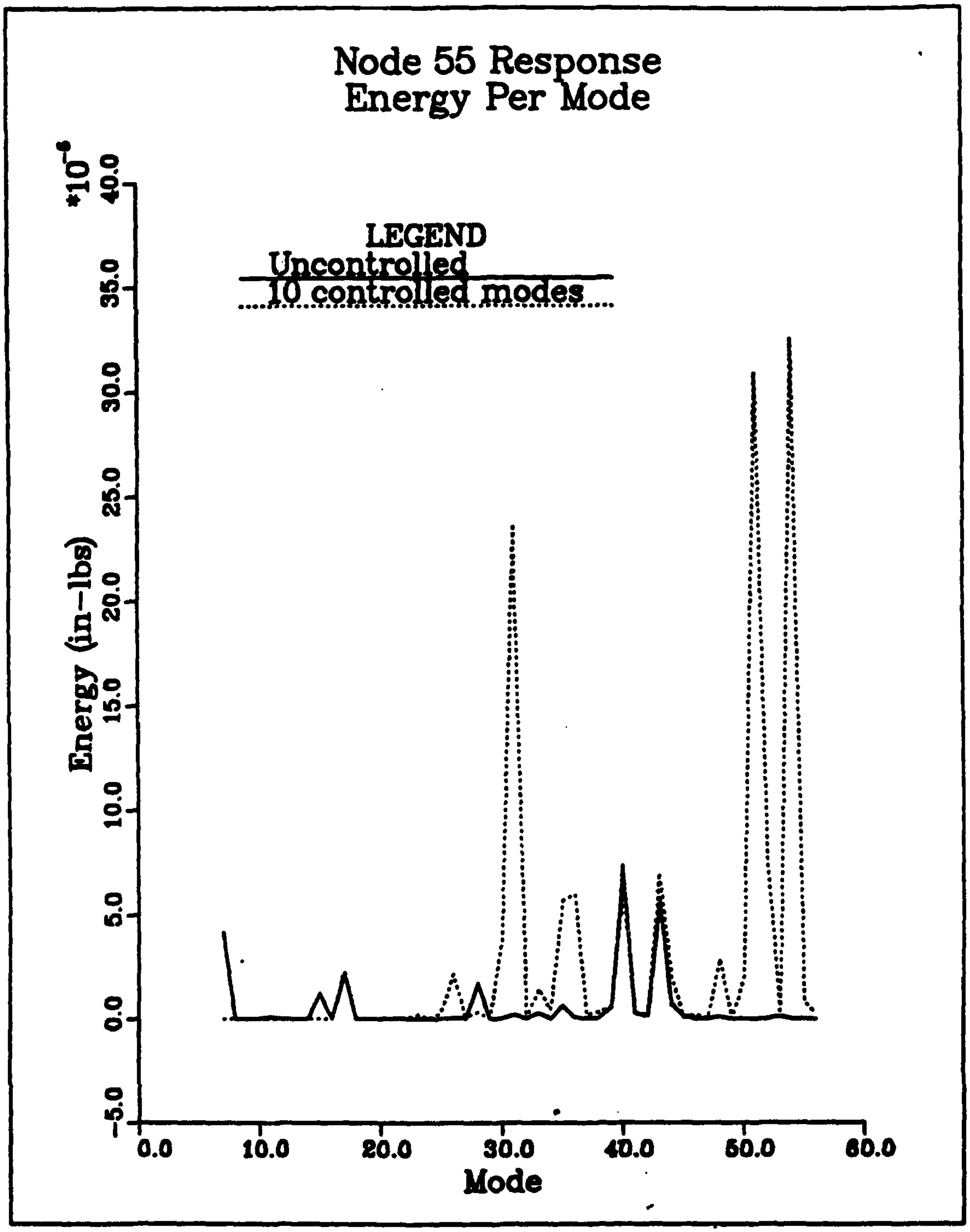

Figure 7. Node 55 Response with 10 Controlled Modes.

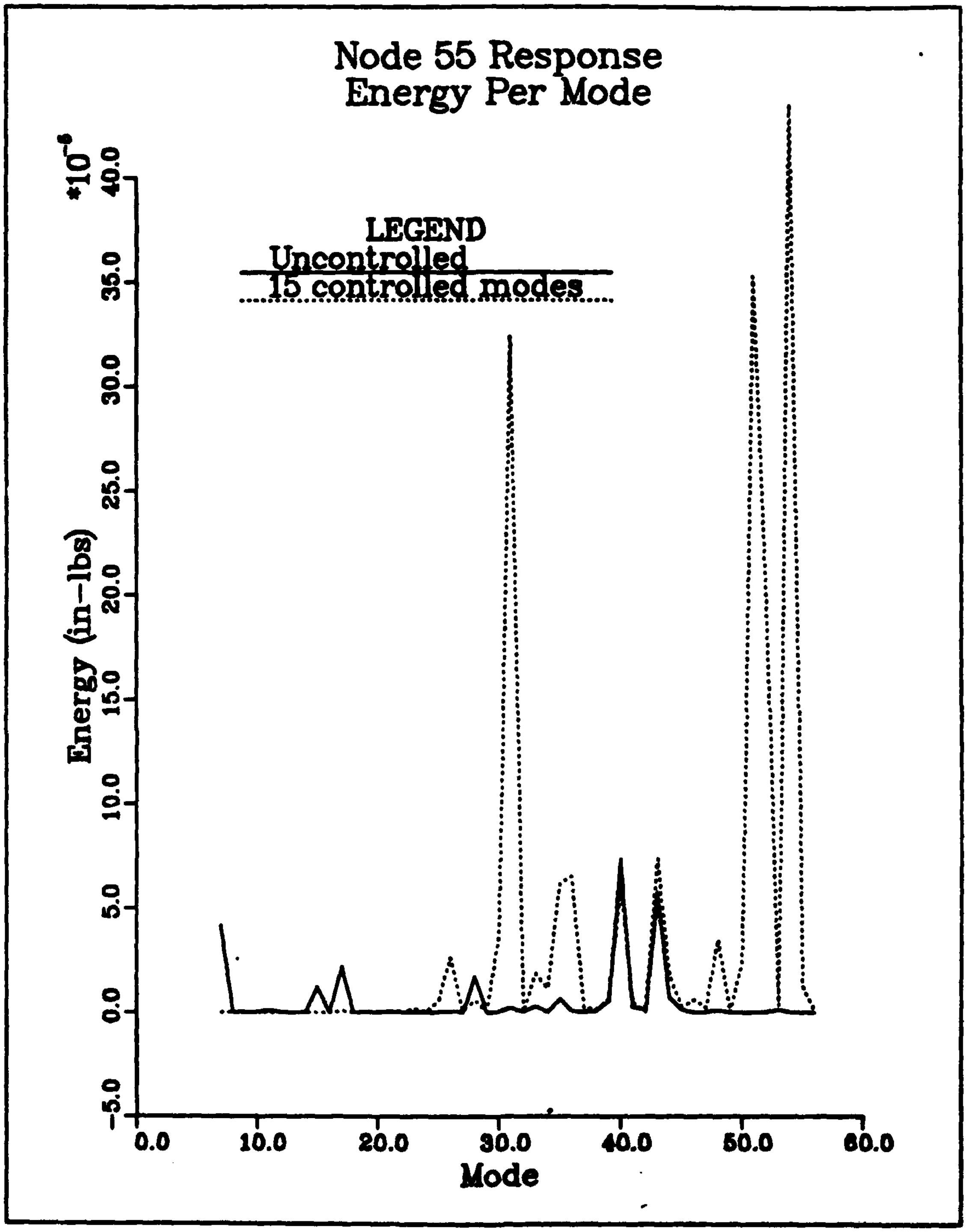

Figure 8. Node 55 Response with 15 Controlled Modes.

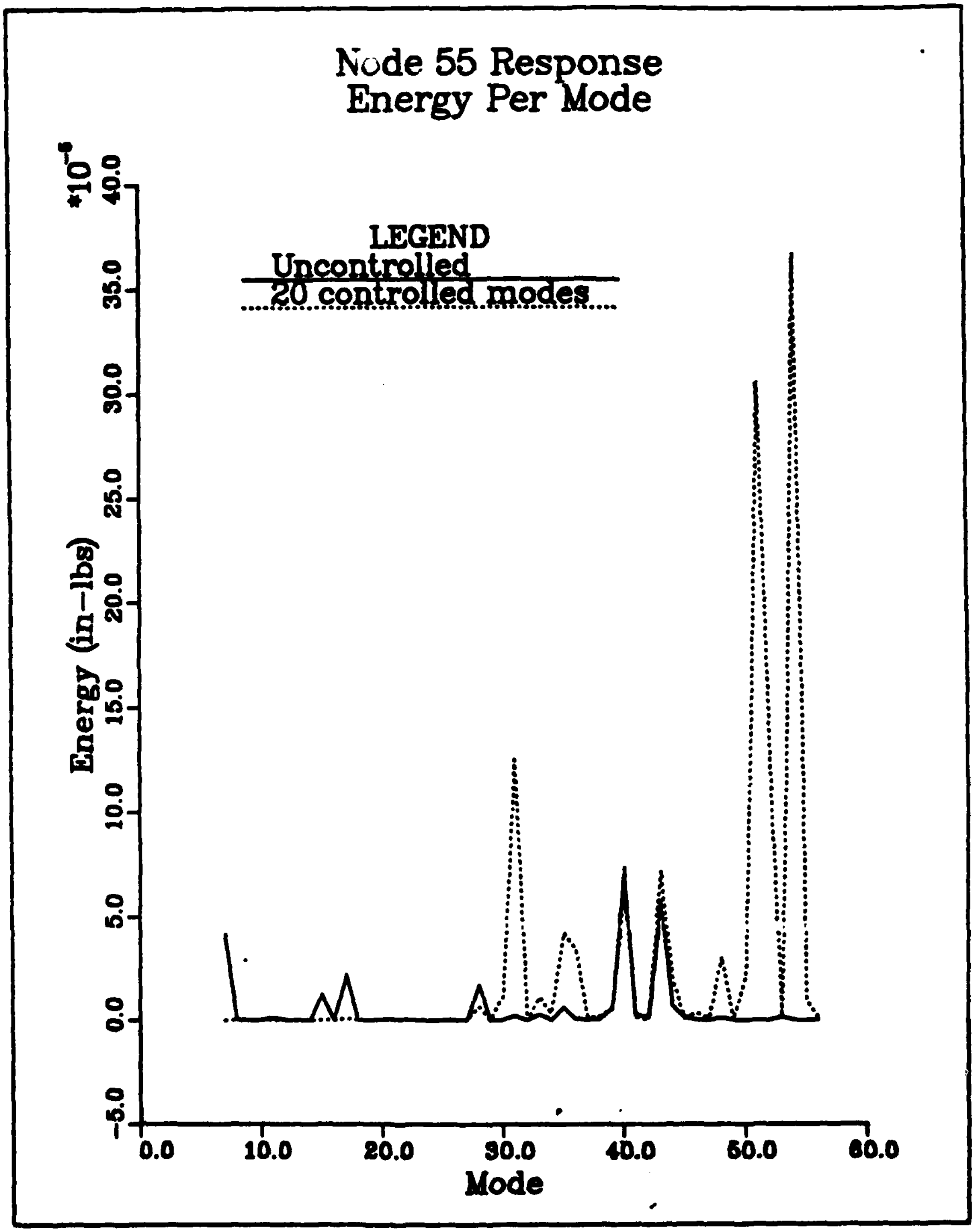

Figure 9. Node 55 Response with 20 Controlled Modes.

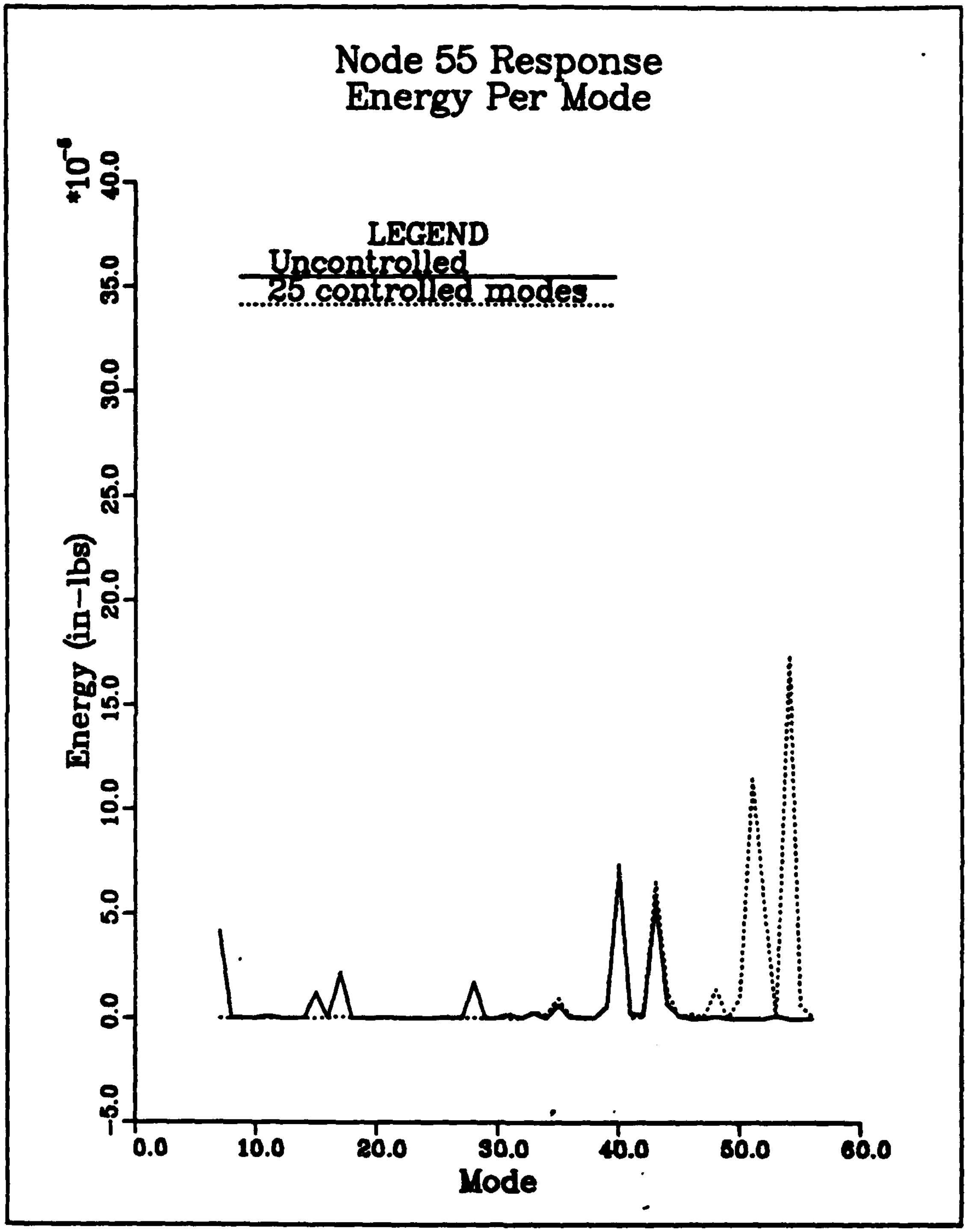

Figure 10. Node 55 Response with 25 Controlled Modes.

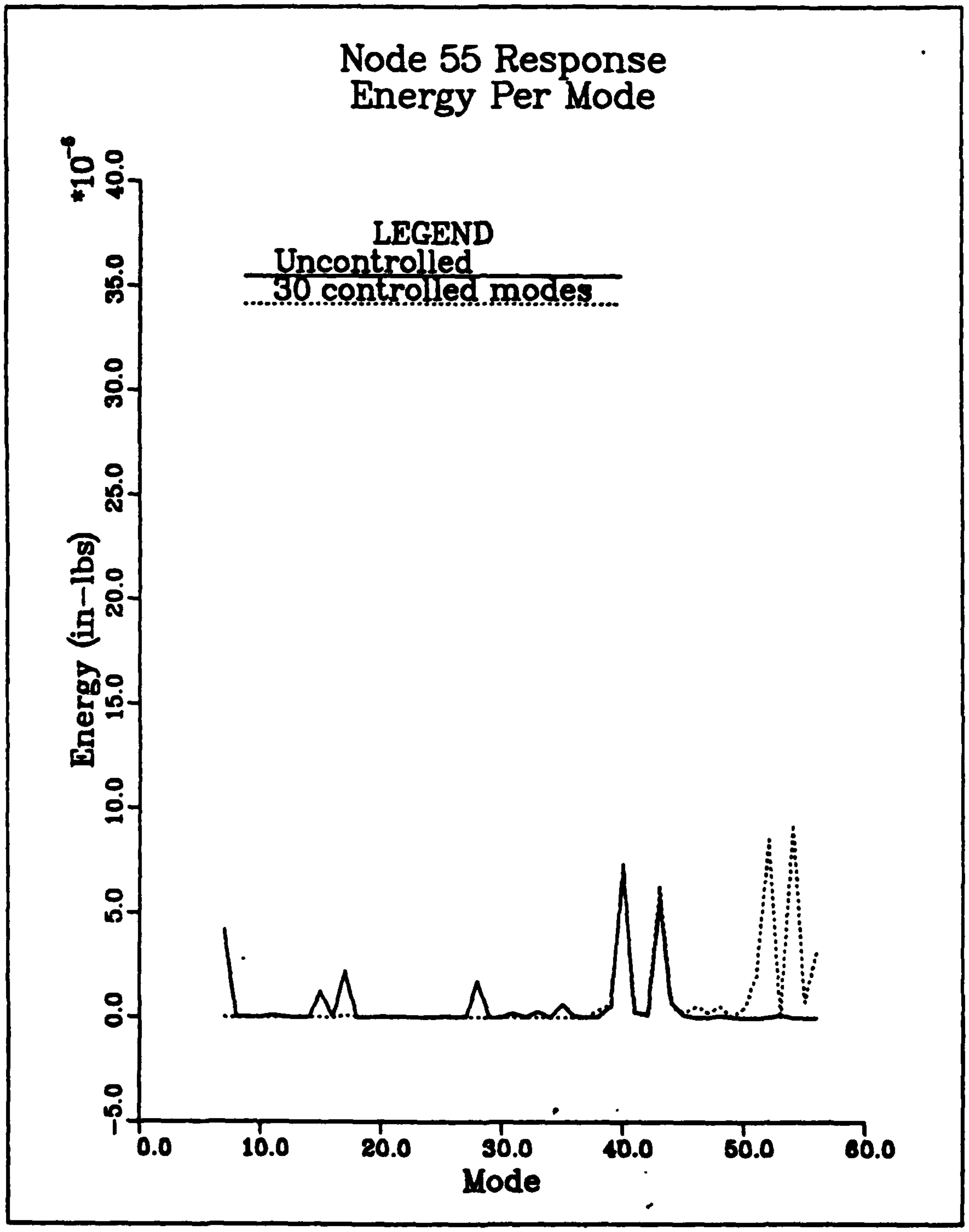

Figure 11. Node 55 Response with 30 Controlled Modes.

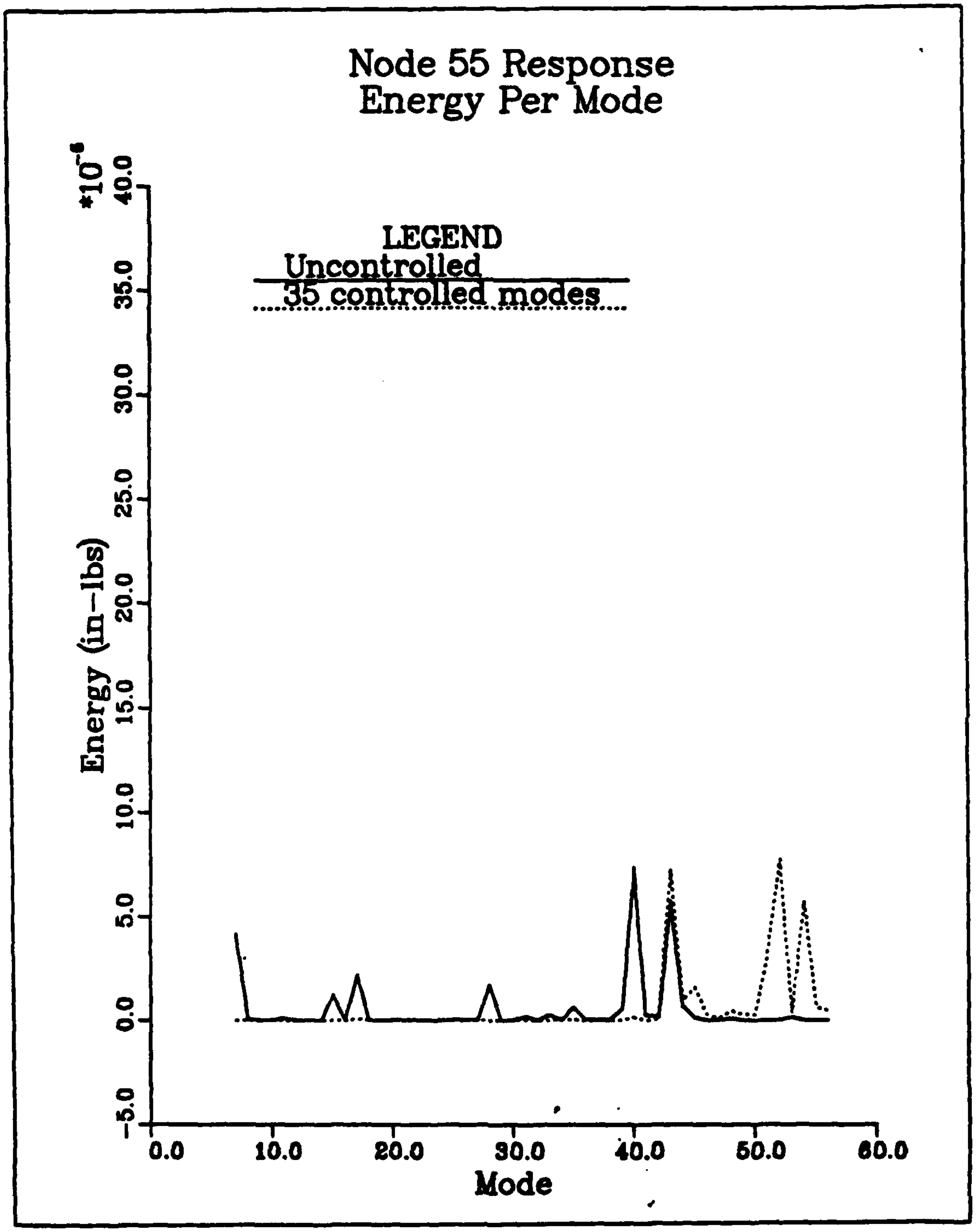

Figure 12. Node 55 Response with 35 Controlled Modes.

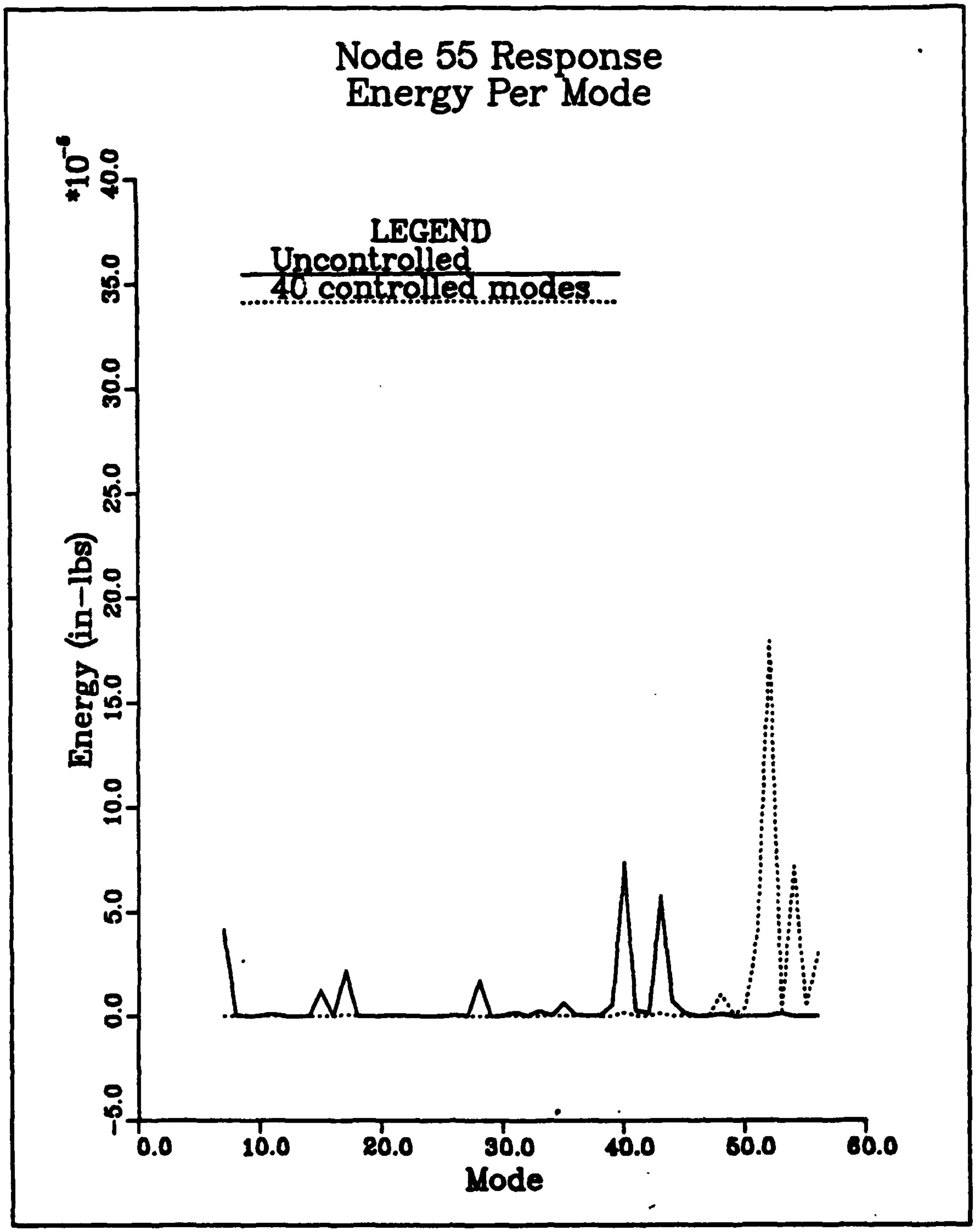

Figure 13. Node 55 Response with 40 Controlled Modes.

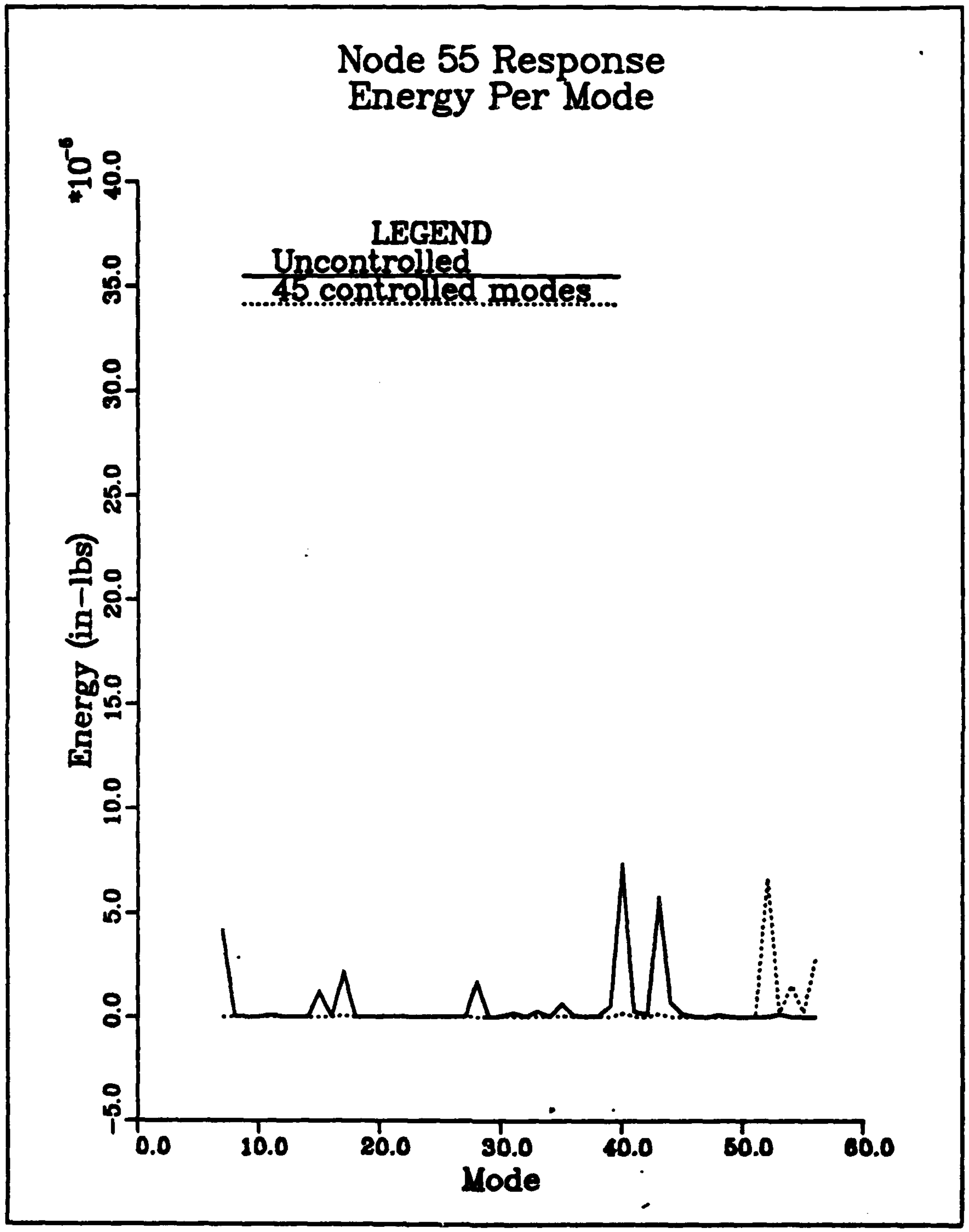

Figure 14. Node 55 Response with 45 Controlled Modes.

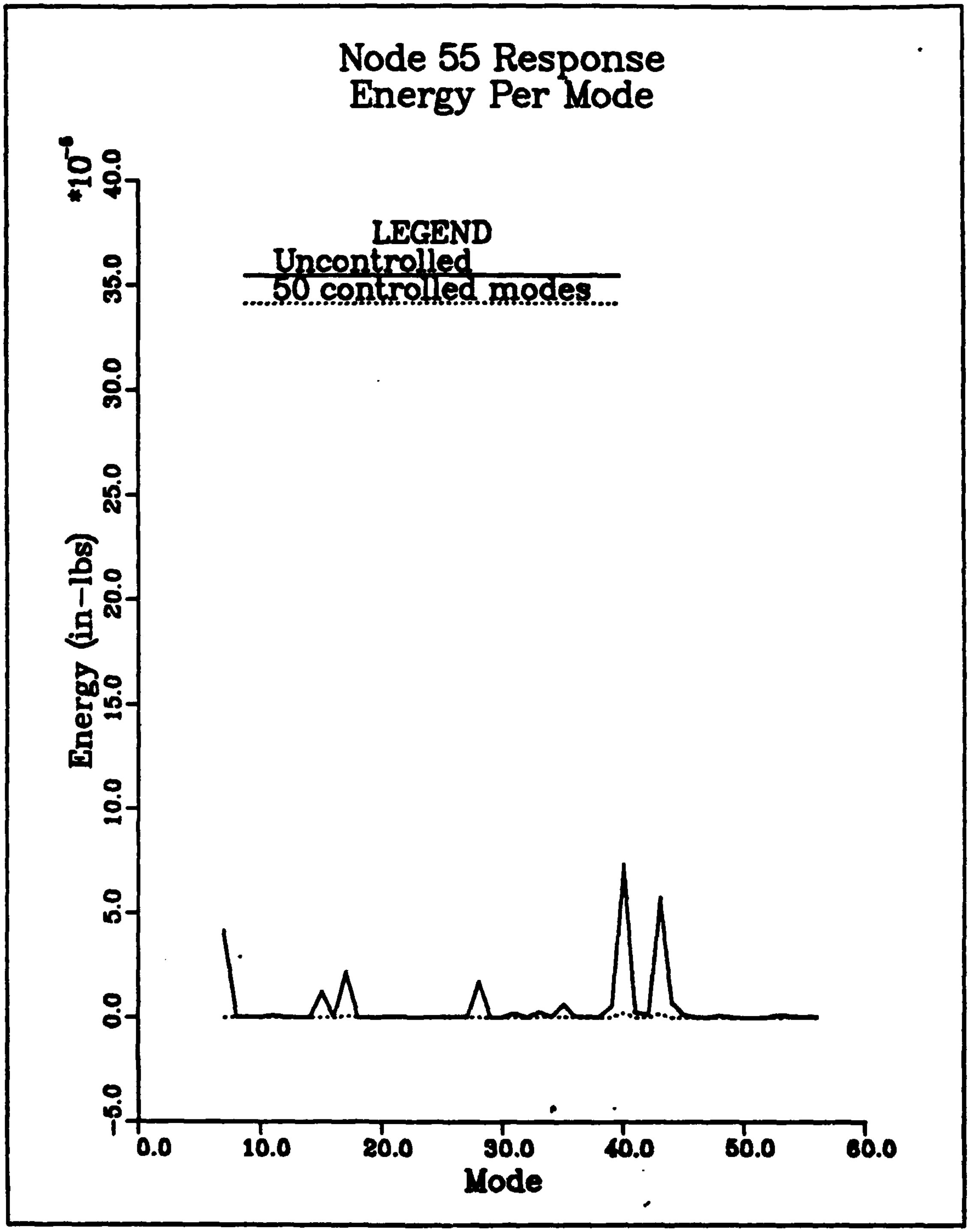

Figure 15. Node 55 Response with 50 Controlled Modes.

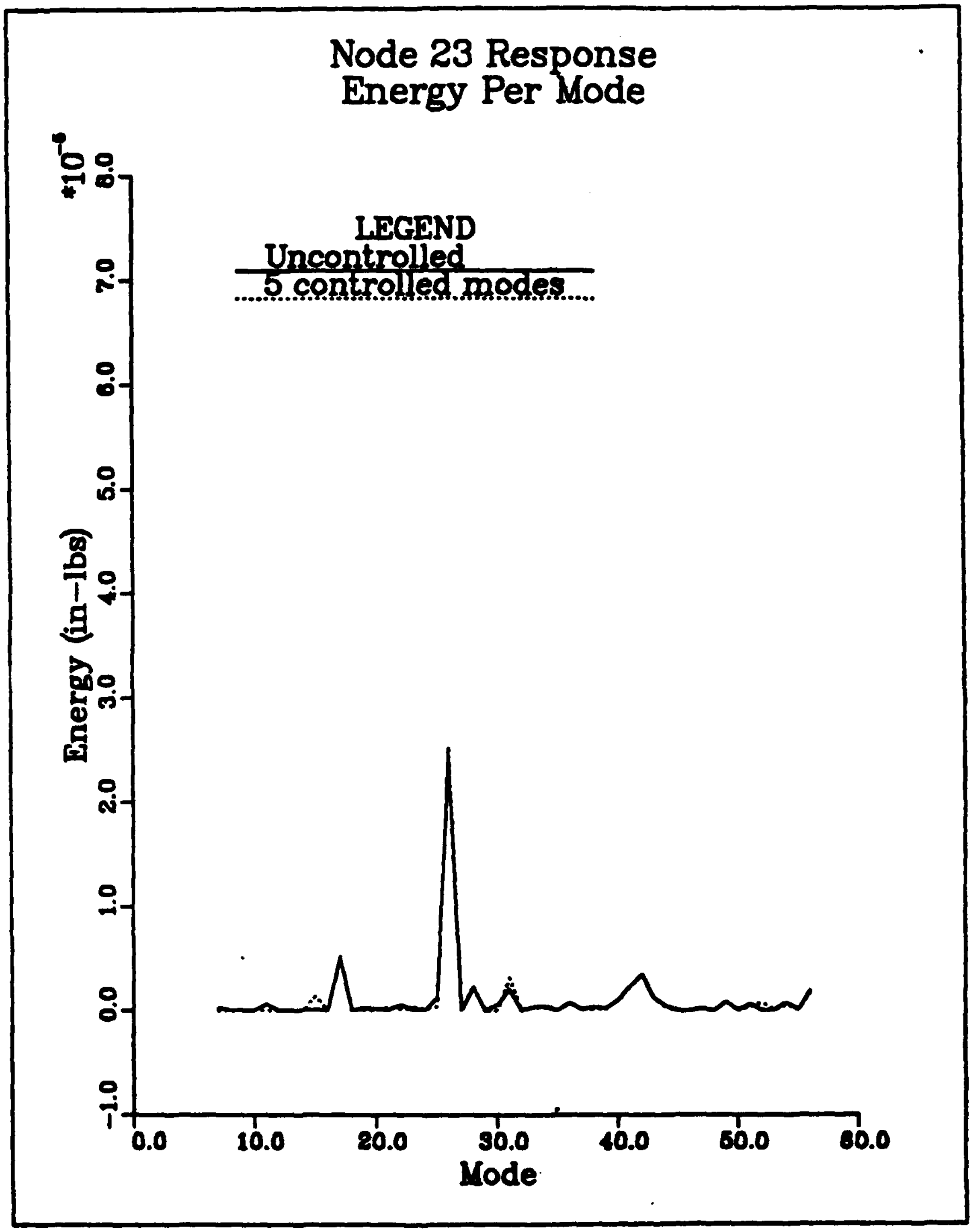

Figure 16. Node 23 Response with 5 Controlled Modes.

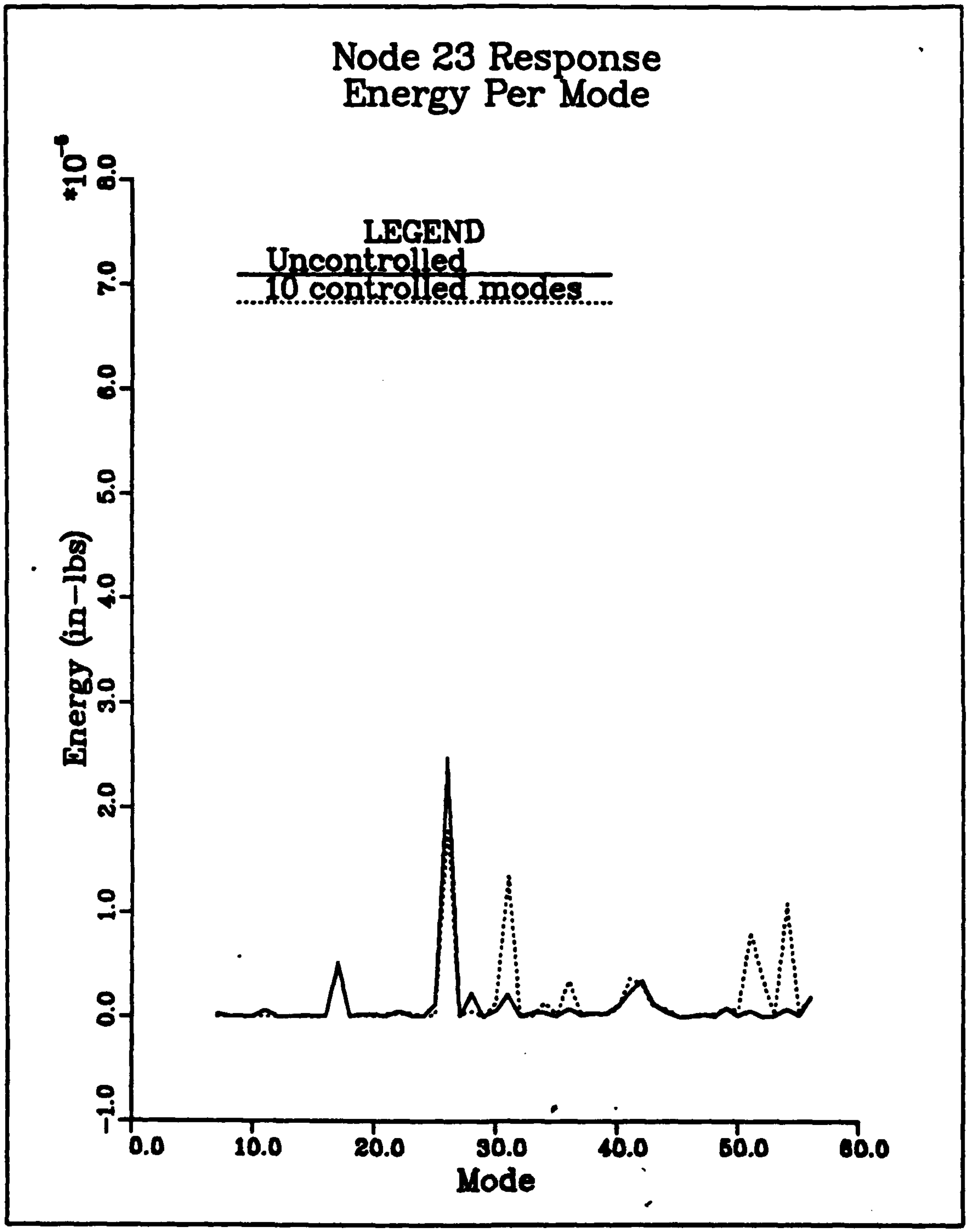

Figure 17. Node 23 Response with 10 Controlled Modes.

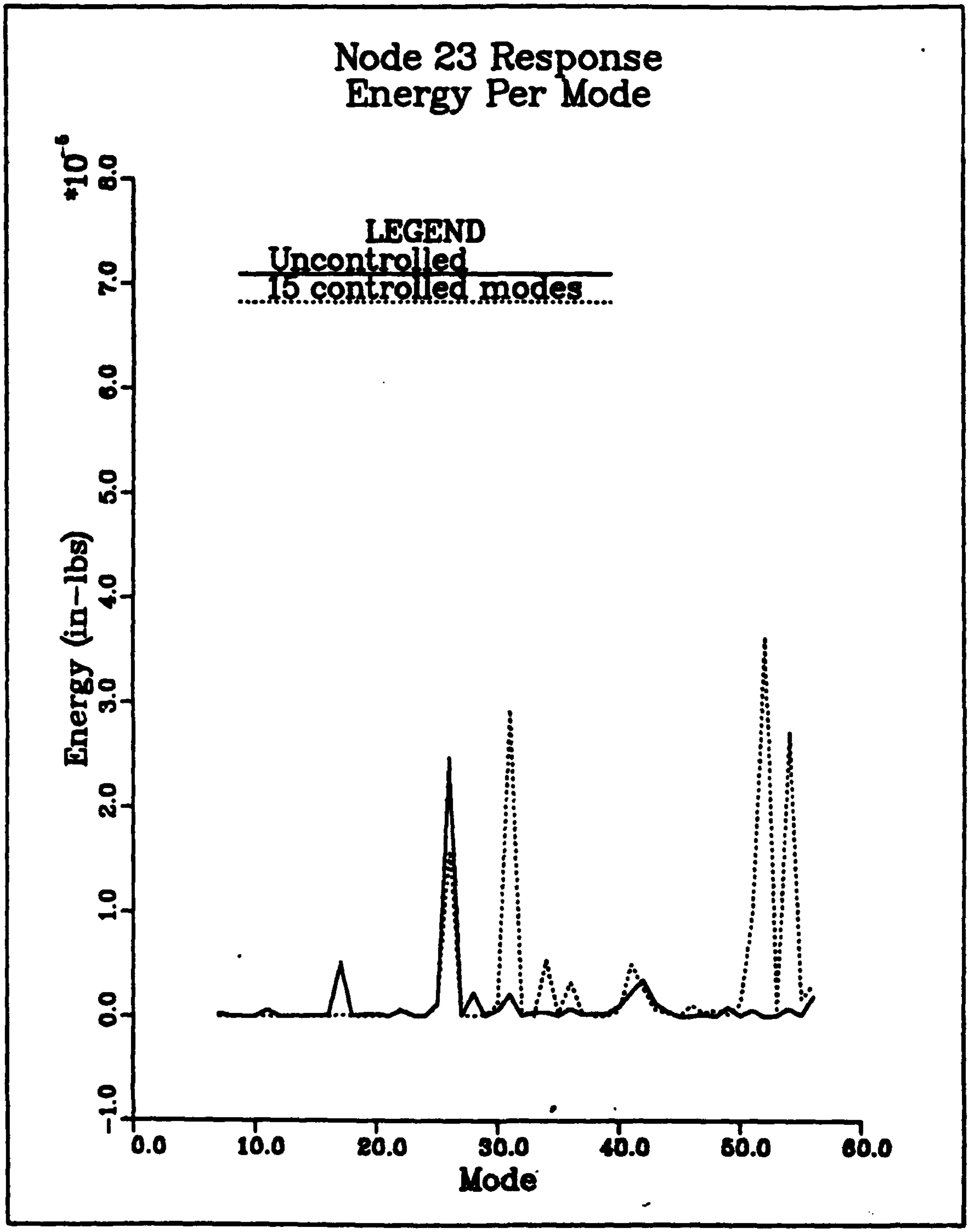

Figure 18. Node 23 Response with 15 Controlled Modes.

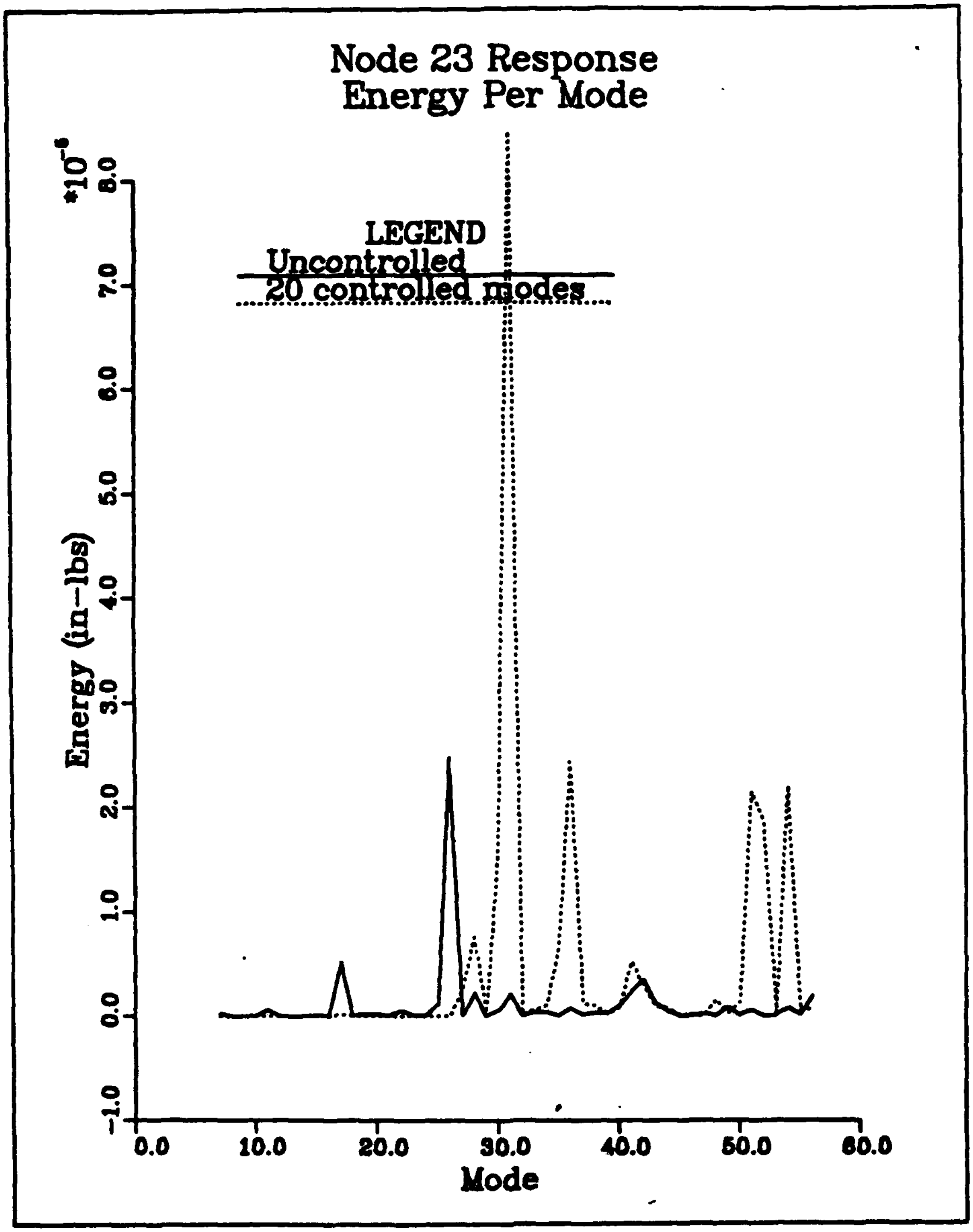

Figure 19. Node 23 Response with 20 Controlled Modes.

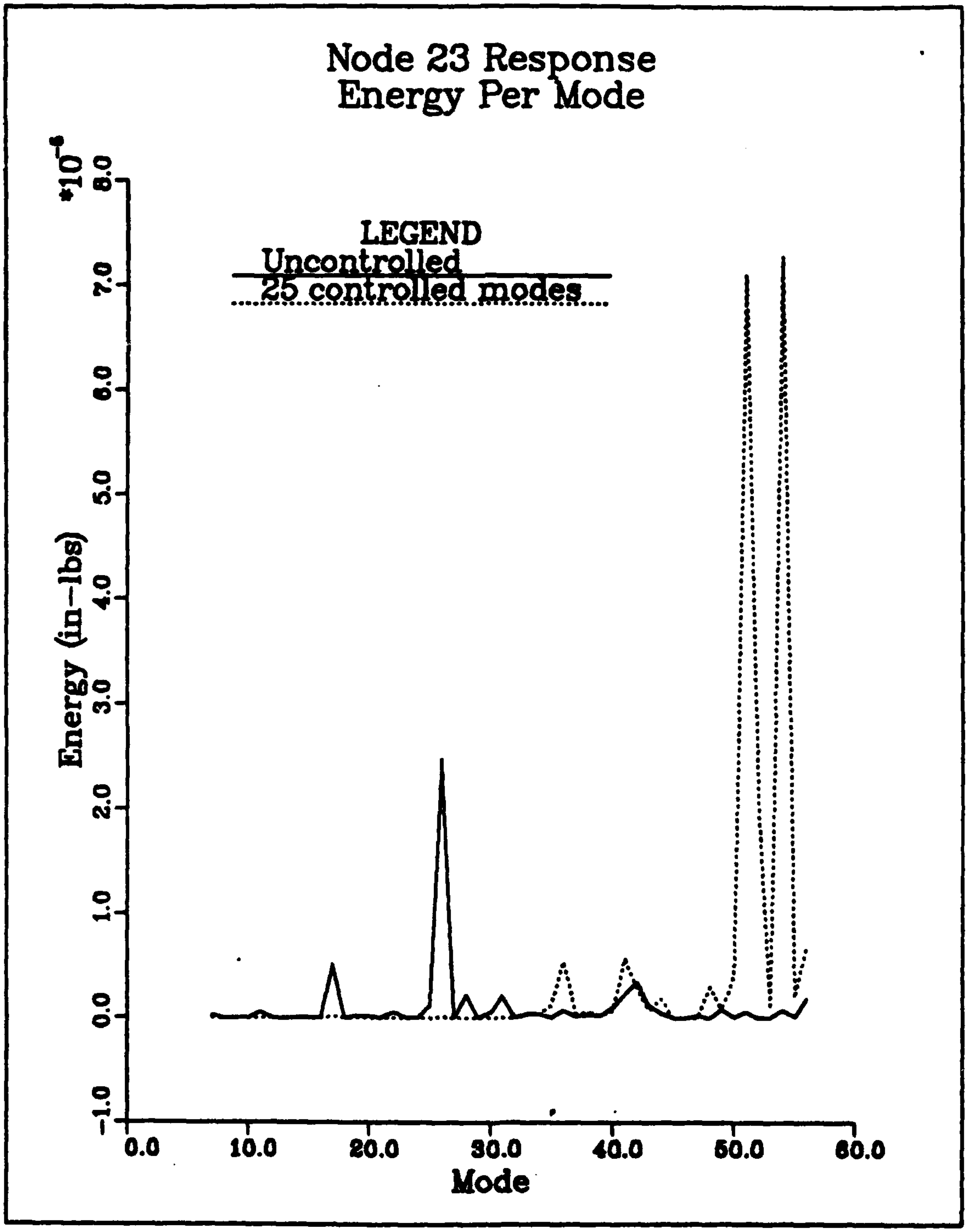

Figure 20. Node 23 Response with 25 Controlled Modes.

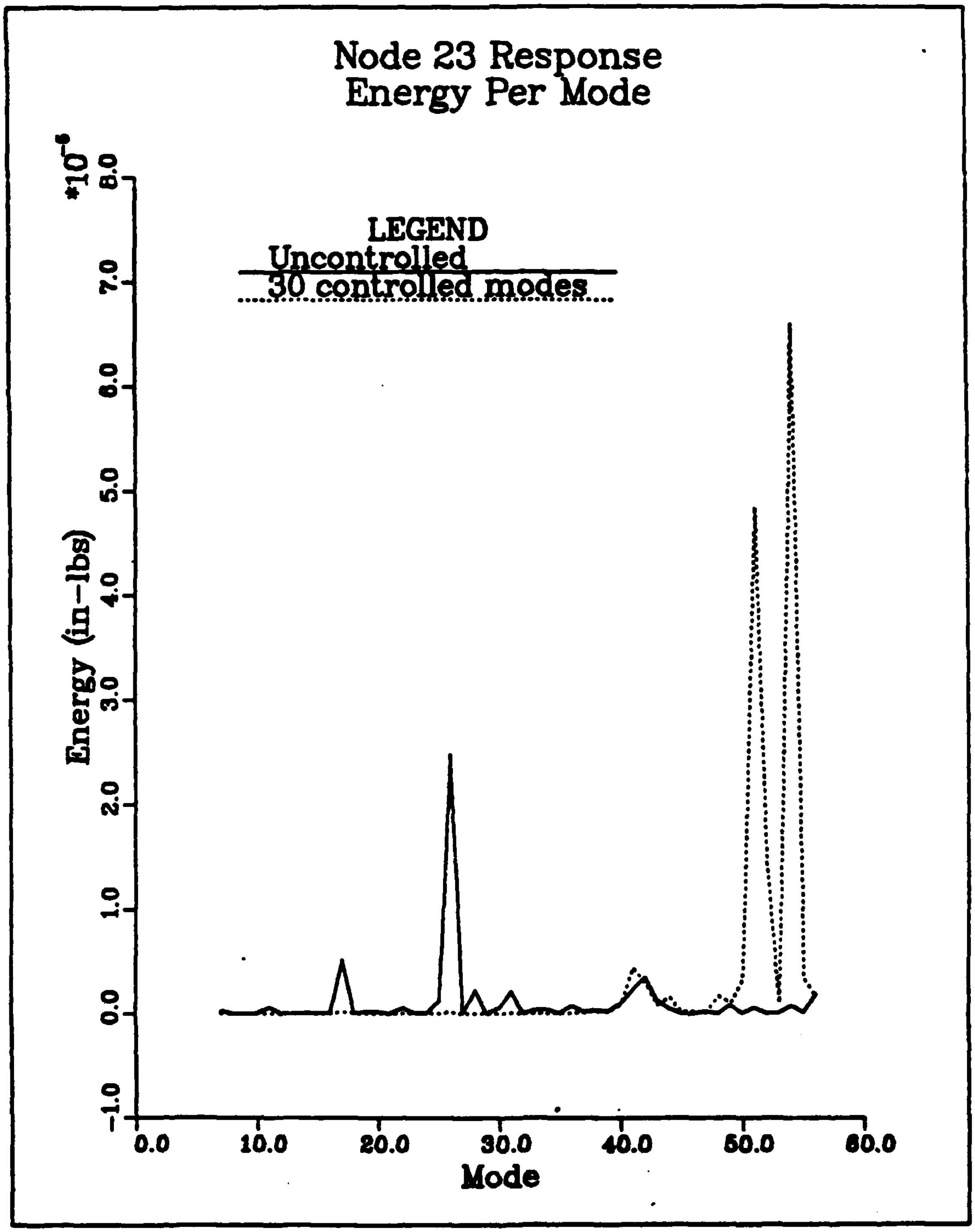

Figure 21. Node 23 Response with 30 Controlled Modes.

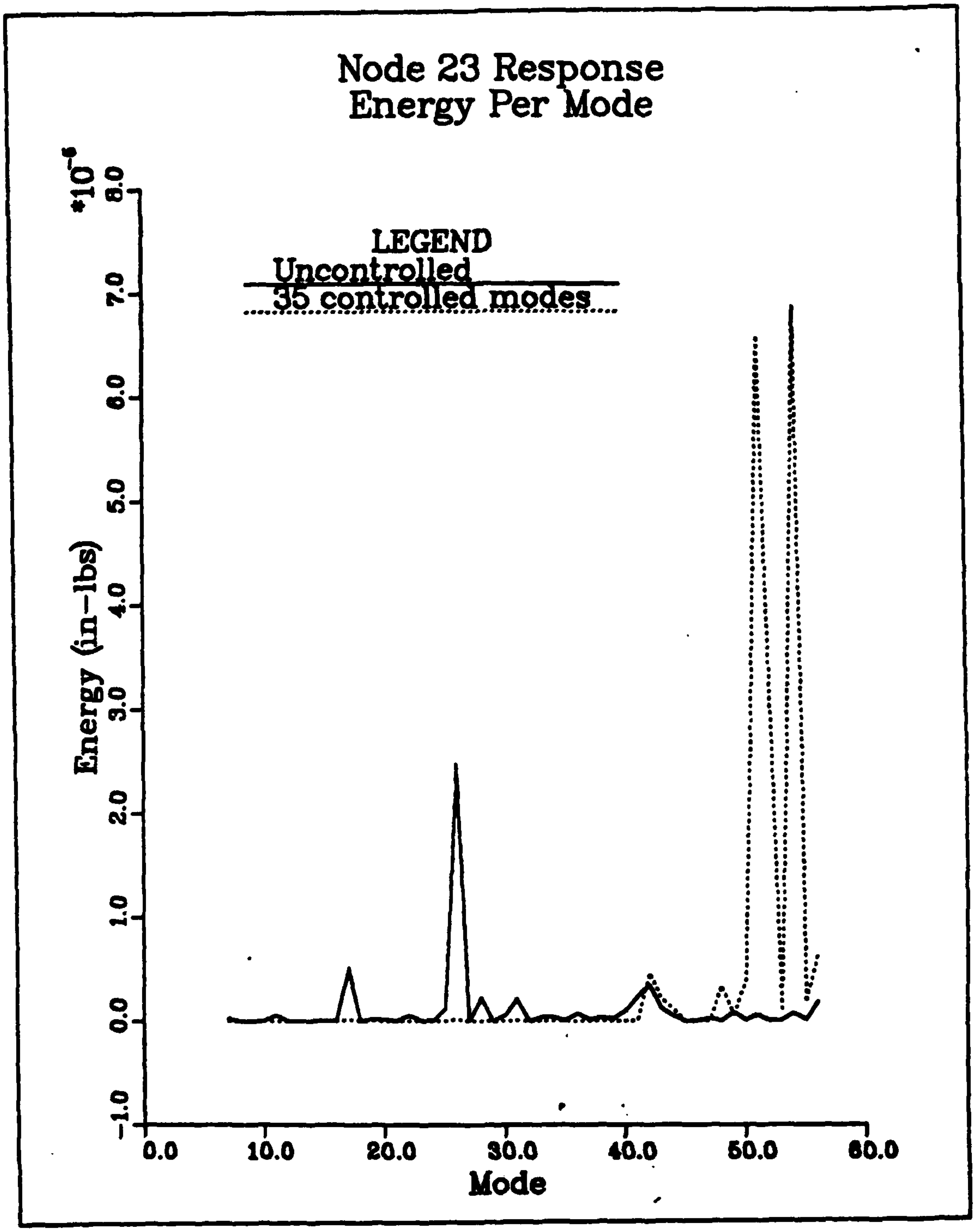

Figure 22. Node 23 Response with 35 Controlled Modes.

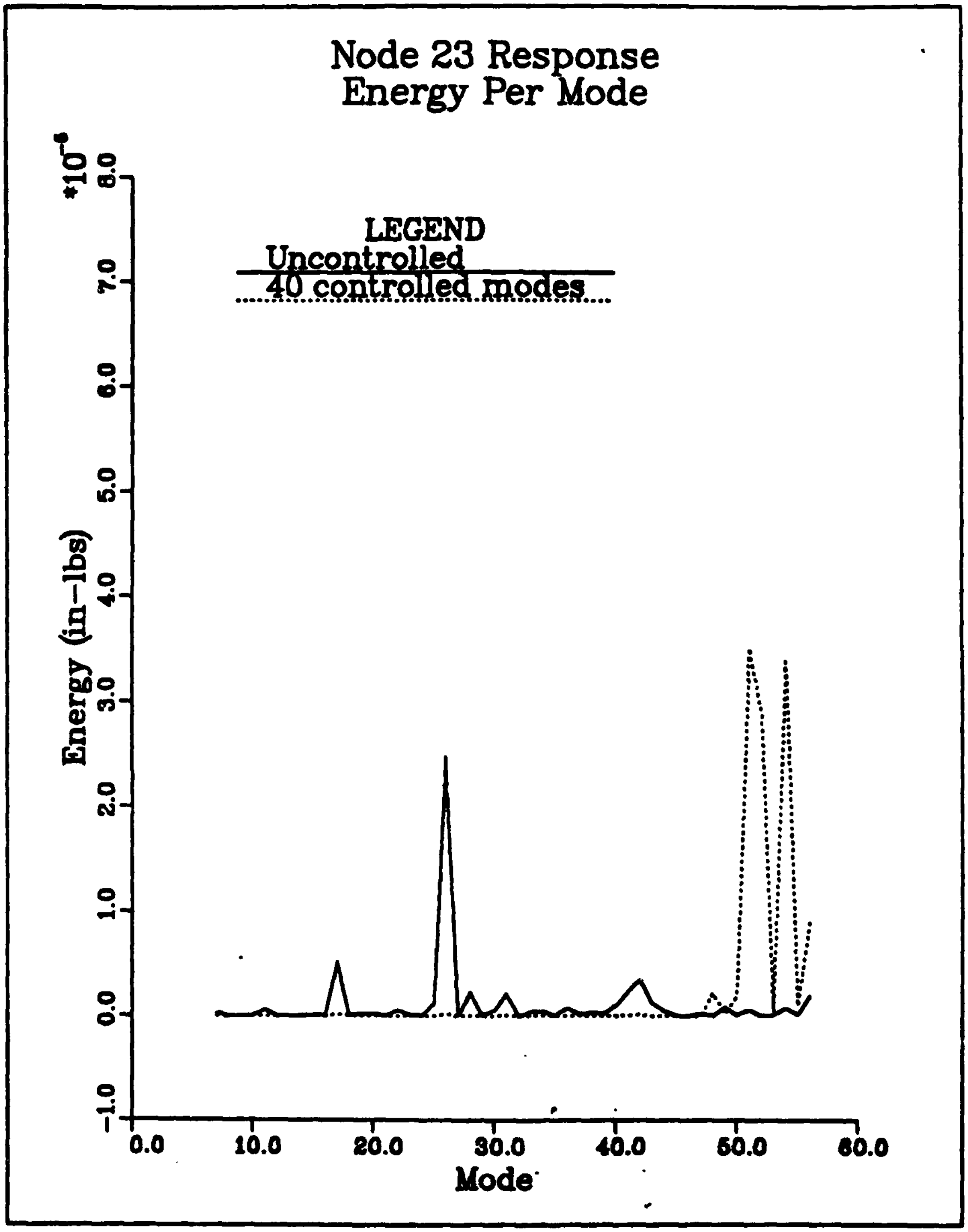

Figure 23. Node 23 Response with 40 Controlled Modes.

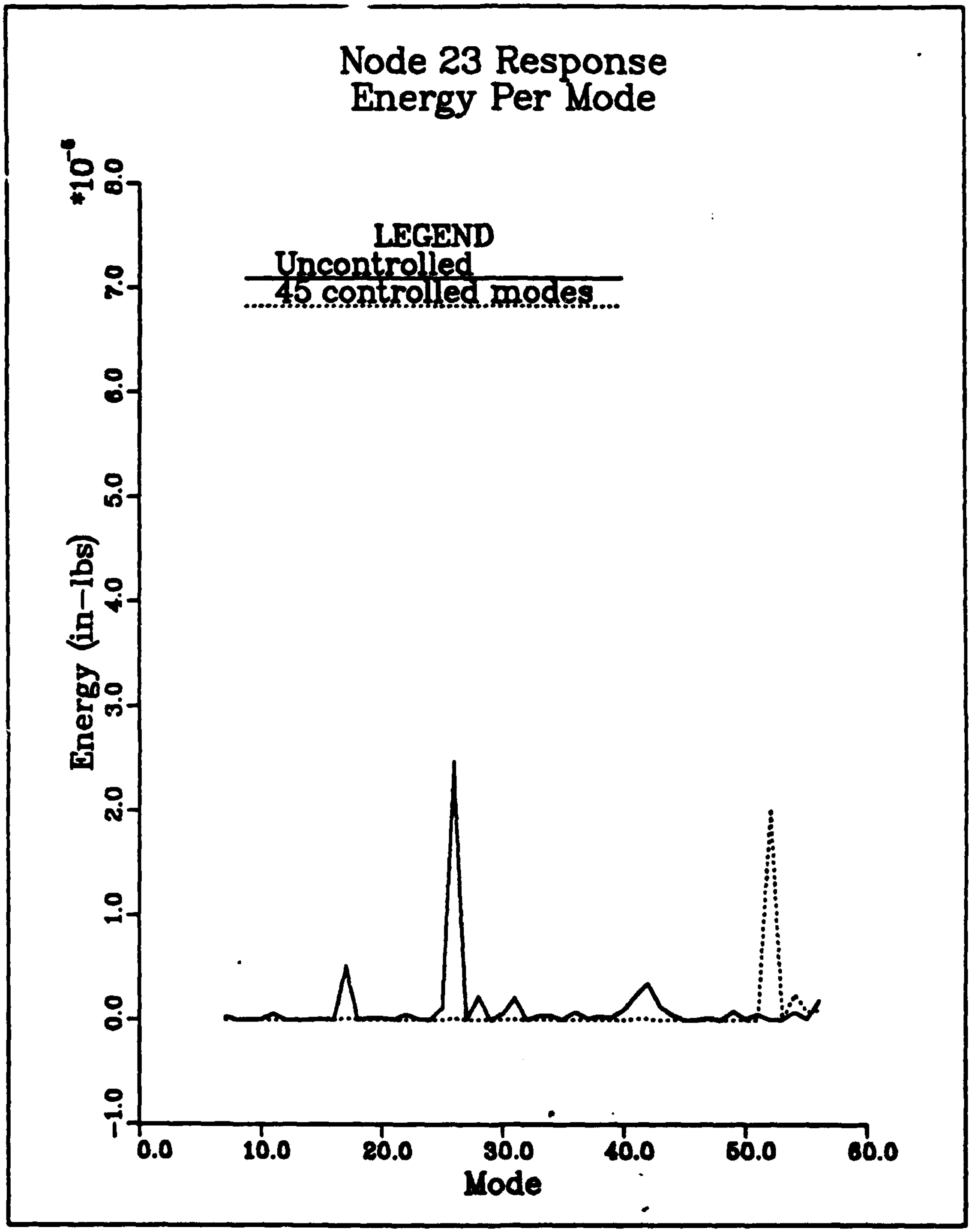

Figure 24. Node 23 Response with 45 Controlled Modes.

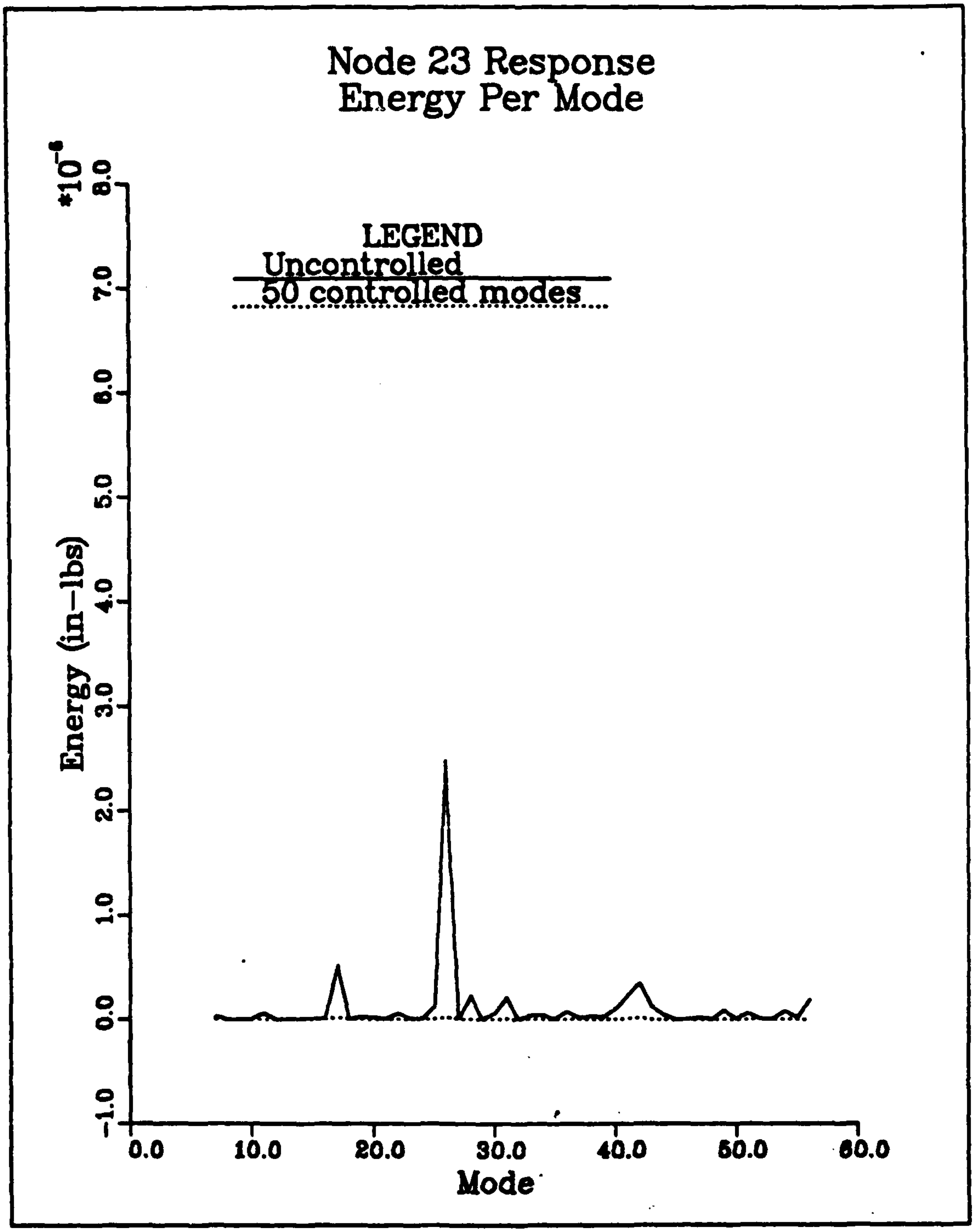

Figure 25. Node 23 Response with 50 Controlled Modes.

The results, total system energy and total energy per mode, have been presented in a graphical format. Total system energy provides a good indication of total performance, but not much more. Figure 4 on page 22 indicates a large number of modes must be controlled to reduce the energy cost to a level below that of an uncontrolled system. An alternative approach to the control problem would include the control of the trouble modes in the formulation of the cost function, which will result in control gains which do not excite these modes.

A better means of observing the system is achieved by presenting the results in the alternate format of energy per mode. Trouble modes identified themselves in Figure 6 on page 25 thru Figure 25 on page 44. These groups of modes were sensitive to any change in the control structure of the space station, and are major contributors to the energy cost until they become directly controlled. Reaching a desirable energy cost will require a reduced order control system of at least 50 modes. This may be prohibitive with regard to complexity, cost and weight.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

A mathematical model of a large space station was developed, several optimal control solutions based on reduced order models were found, and the effects of reduced order modeling on the control of a large space structure have been presented in this thesis. It was shown that a straightforward approach to reduced order control can be achieved for a space station. The negative aspect of this approach is that the number of modes required to reach an acceptable level of control is considered to be excessive from a cost and weight viewpoint.

The existence of certain trouble modes was observed. These mode groups contributed significantly to the cost of control until the control model became large enough to directly control these modes. Also, it was demonstrated that these mode groups remained troublesome regardless of the location of the disturbance.

B. RECOMMENDATIONS

Based on the results of this work, there are several additional areas that may be investigated with respect to the effects of reduced order control on a space structure. Some of these areas are:

- to develop a cost function that considers control of the trouble modes in the optimal gain solution
- to examine the effects of notch filtering to supress excitation of the trouble modes
- to utilize Karhunen-Loeve expansion methods for reduced order control modeling.

Research in these areas may produce results more favorable to solving the control problems of a large space structure.

APPENDIX A. SPACE STRUCTURE SIMULATION PROGRAM

```
C
C     ************************************************************
C     *****                                                 *****
C     *****      SPACE STRUCTURE SIMULATION PROGRAM         *****
C     *****                                                 *****
C     ************************************************************
C
C
C     ************************************************************
C     *****              VARIABLE DECLARATIONS               *****
C     ************************************************************
C
      EXTERNAL EXCMS,RICDSD
      CHARACTER*6 NAM
      CHARACTER*1 AGAIN,CORECT,RAGAIN
      INTEGER NODE,MODE,KQ,EMODE,SMODE,CTADJ,CT,CF,KA,I,K,M
      INTEGER LOOP,PRNT,MODAL,COUNT,PRNTG,NF,NG,NH,NZ,V
      INTEGER IPVS(100),ITYPE(200)
C
      REAL RTOTAL,RMODEN(7:100)
      REAL*8 PHII(2,2,100),GAMMA(2,100),EGT,GMA,WN,W1,X1T,X2T
      REAL*8 PHI(188,188),B(188,3),BN(188,3),R(3,3),RR(3,3)
      REAL*8 RINV(3,3),RRINV(3,3),X1(7:100),X2(7:100),MODEN(7:100)
      REAL*8 COSW1T,SINW1T,COST,CNTCST,ENERGY,TOTCST,RM
      REAL*8 TCX,TCY,TCZ,DAMP,SAMPT,PI,SUM1,SUM2,SUM3,SUMC
      REAL*8 TNX,TNY,TNZ,IMPX,IMPY,IMPZ,IMPLSX,IMPLSY,IMPLSZ
      REAL LAMA(100),UGVEX(100,3),RNODE,RMODE,MIN,TIME,SAMPTM
      REAL UG69(100,3),UG23(100,3),UG55(100,3)
      REAL*8 H(100,100),G(100,100),L(3,100),BT(3,100)
      REAL*8 Z(200,200),W(200,200),ER(200),F(100,100),EI(200)
      REAL*8 SCALE(200),TEMP(100,3),TEMP1(3,100),WORK(100)
C
C     ************************************************************
C     *****              VARIABLE DEFINITIONS                *****
C     ************************************************************
C
C     LAMA = VECTOR OF THE SQUARE OF THE NATURAL FREQUENCIES
C     UGVEX = MODE POSITONS AND SLOPES OF THE NODAL POINTS
C     PHII = TRANSITION MATRICIES FOR EACH MODE
C     PHI = BLOCK DIAGONAL STATE TRANSITION MATRIX CONSISTING OF
C           THE INDIVIDUAL PHII MATRICIES
C     GAMMA = INPUT TRANSITION MATRIX
C     B = INPUT MATRIX OF GAMMA AND CONTROL NODE SLOPES
C     BN = NOISE INPUT MATRIX OF GAMMA AND NOISE NODE SLOPES
C     DAMP = DAMPING FACTOR
C     SAMPT = SAMPLING TIME
C     IMPLSE = IMPULSE INPUT FUNCTION
C     TCX, TCY, TCZ = CONTROL TORQUE VALUES
C     IMPX, IMPY, IMPZ = AXIS IMPULSE NOISE VALUES
C     ENERGY = SYSTEM ENERGY COST VALUE FOR A GIVEN POINT IN TIME
C     CNTCST = SYSTEM CONTROL COST VALUE FOR A GIVEN POINT IN TIME
```

```
C      COST = TOTAL SYSTEM COST VALUE FOR A GIVEN POINT IN TIME
C      TOTCST = SYSTEM COST SUMMED OVER ALL TIME
C      MIN = NUMBER OF MINUTES SYSTEM WILL BE OBSERVED
C
C      **********      SAMPLE OF SPACE EXEC FILE      **************
C
C      THIS FILE MUST BEGIN IN COLUMN 1 AND RUN IN THE FOLLOWING
C      SEQUENCE FOR THE INITIAL RUN OF THE PROGRAM:
C
C              FORTVS SPACE            (COMPILES PROGRAM)
C              SPACE                   (EXECUTES EXEC FILE)
C              LOAD SPACE (START       (LOADS AND EXECUTES PROGRAM)
C
C      SUBSEQUENT PROGRAM RUNS CAN ELIMINATE "FORTVS SPACE" IF NO
C      CHANGES HAVE BEEN MADE TO THE PROGRAM AND CAN ELIMINATE
C      RUNNING THE EXEC FILE.
C
C      FI 4 DISK THESIS INPUT A (PERM
C      FI 30 DISK X1 OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 31 DISK MODENG OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 32 DISK TORQUE OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 33 DISK ENERGY OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 34 DISK MDECST OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 35 DISK COUNT OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 40 DISK UTILITY OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 41 DISK RUN OUTPUT A (RECFM F BLOCK 80 PERM
C      FI 42 DISK NODE69 INPUT A (RECFM F BLOCK 80 PERM
C      FI 43 DISK NODE23 INPUT A (RECFM F BLOCK 80 PERM
C      FI 44 DISK NODE55 INPUT A (RECFM F BLOCK 80 PERM
C
C      THE THESIS INPUT FILE CONTAIN THE NATURAL FREQUENCIES OF THE
C      SYSTEM (LAMA VECTOR).  MODAL SLOPES FOR THE DESIRED NODES ARE
C      CONTAINED IN THE INPUT FILES NODE69, NODE55 AND NODE23.  THE
C      REMAINING FILES IN THE EXEC CAN BE STRUCTURED TO HANDLE OUTPUT
C      AS DESIRED.

C      ***********************************************************
C
       PI = 4.0D0 * ATAN(1.0D0)
       SAMPT = 0.0
       DAMP = 0.0
       MODAL = 0
       NF = 100
       NG = 100
       NH = 100
       NZ = 200
C
C      *****  NUMBER OF MINUTES THE SYSTEM WILL BE OBSERVED  *****
C
       MIN = 120.0
C
C      ***********************************************************
C      *****             SET LENGTH OF MODAL MODEL           *****
C      ***********************************************************
C
       CALL EXCMS ('CLRSCRN')
```

```
      WRITE (6,1008)
      WRITE (6,*) '                        SET MAXIMUM OPERATING MODE NUMBER'
      READ *, MODAL
C
C     **********************************************************
C     *****               READ LAMA MATRIX                *****
C     **********************************************************
C
      CALL EXCMS ('CLRSCRN')
C
      READ(4,1001) NAM
      READ(4,1002)(LAMA(I),I=1,100)
C
C     **********************************************************
C     *****              SCREEN INTERACTION                *****
C     **********************************************************
C
500   CALL EXCMS ('CLRSCRN')
C
C     ************       STARTING MODE NUMBER      **************
C
      WRITE (6,1004)
10    WRITE (6,*) 'ENTER THE STARTING MODE NUMBER(MUST BE  7  OR GREATER
     +:  '
      READ *, RMODE
      IF(MOD(RMODE,1.0).NE.0) THEN
         WRITE (6,*)'     MODE SELECTION MUST BE AN INTEGER VALUE.    RE
     +ENTER'
         GOTO 10
      ENDIF
      SMODE = INT(RMODE)
      IF ((SMODE.LT.7).OR.(SMODE.GT.100))THEN
        WRITE (6,*)'MODE CHOICES ARE LIMITED TO  7 THRU 100!!  REENTER.'
        GOTO 10
      ENDIF
C
C     **********    NUMBER OF MODES USED FOR CONTROL     ********
C
15    WRITE (6,*) 'ENTER THE NUMBER OF MODES FOR CONTROL.  THE CHOICE MU
     +ST BE FROM 1 TO 94:     '
      READ *, RMODE
      IF(MOD(RMODE,1.0).NE.0) THEN
         WRITE (6,*)'     MODE SELECTION MUST BE AN INTEGER VALUE.    RE
     +NTER'
         GOTO 15
      ENDIF
      MODE = INT(RMODE)
      IF ((MODE.LT.1).OR.(MODE.GT.94))THEN
        WRITE (6,*)'MODE CHOICES ARE LIMITED TO  1 THRU 94!!  REENTER.'
        GOTO 15
      ENDIF
      EMODE = SMODE + MODE - 1
C
C     *************     NOISE INPUT POSITION      *****************
C
      CALL EXCMS ('CLRSCRN')
```

```
      WRITE (6,1004)
      IMPX = 0.0D0
      IMPY = 0.0D0
      IMPZ = 0.0D0
20    WRITE (6,*) 'ENTER THE NOISE INPUT LOCATION.  THE CHOICES ARE 23,
     +55 OR 69.  '
      READ *, RNODE
      IF(MOD(RNODE,1.0).NE.0) THEN
         WRITE (6,*)'     NODE SELECTION MUST BE AN INTEGER VALUE.     RE
     +NTER'
         GOTO 20
      ENDIF
      NODE = INT(RNODE)
      IF ((NODE.LT.1).OR.(NODE.GT.114))THEN
        WRITE (6,*)'NODE CHOICES ARE LIMITED TO  1 THRU 114!!  REENTER.'
        GOTO 20
      ENDIF
      WRITE (6,*) 'SELECT THE NUMBER OF THE NOISE INPUT AXIS  '
      WRITE (6,*) ' '
      WRITE (6,*) '       0    NO NOISE INPUT'
      WRITE (6,*) '       1    X AXIS INPUT  '
      WRITE (6,*) '       2    Y AXIS INPUT  '
      WRITE (6,*) '       3    Z AXIS INPUT  '
      WRITE (6,*) '       4    INPUT ON ALL AXIES '
      READ *, AXIS
C
C     *************           R MATRIX VALUE        ****************
C
      CALL EXCMS ('CLRSCRN')
      WRITE (6,1004)
      WRITE (6,*) 'ENTER YOUR INITIAL  R  VALUE:  '
      READ *, RM
C
C     *************           SAMPLING TIME           **************
C
25    WRITE (6,*) 'ENTER SAMPLING TIME (MUST BE EQUAL TO OR LESS THAN 0.
     +04 SEC):  '
      READ *, SAMPT
         SAMPTM = ((2.0D0*PI)/SQRT(LAMA(100)))/2.0D0
         IF (SAMPT.GE.SAMPTM) THEN
              WRITE(6,*)'   ERROR:  SAMPLING TIME MUST BE LESS THAN OR E
     +QUAL TO'
              WRITE(6,1005) SAMPTM
              GOTO 25
         ENDIF
C
C     ***************          DAMPING FACTOR         **************
C
30    WRITE (6,*) 'ENTER DAMPING FACTOR  D:    '
      READ *, DAMP
      IF ((DAMP.LT.0).OR.(DAMP.GT.1.0))THEN
        WRITE (6,*)'DAMPING FACTOR IS LIMITED TO 0 THRU 1.0  -  REENTER'
        GOTO 30
      ENDIF
C
```

```
C      *************       VALUES CORRECT ????       ***************
C
       CALL EXCMS ('CLRSCRN')
       WRITE (6,1004)
520    WRITE (6,*) 'ARE THESE VALUES CORRECT?    (Y/N)      'CAPS ONLY '
       WRITE (6,*) ' '
       WRITE (6,706) SMODE
       WRITE (6,707) MODE
       WRITE (6,708) NODE
       WRITE (6,709) RM
       WRITE (6,710) SAMPT
       WRITE (6,711) DAMP
706    FORMAT (' ','             STARTING MODE NUMBER:   ',I2)
707    FORMAT (' ','             NUMBER OF MODES SCANNED:   ',I2)
708    FORMAT (' ','             NOISE INPUT NODE:   ',I3)
709    FORMAT (' ','             INITIAL R VALUE:   ',E12.4)
710    FORMAT (' ','             SAMPLING TIME:   ',E12.4)
711    FORMAT (' ','             DAMPING FACTOR:   ',E12.4)
       READ(*,1010) CORECT
          IF(CORECT.EQ.'Y') THEN
               GOTO 510
          ELSEIF(CORECT.EQ.'N') THEN
               GOTO 500
          ELSE
               WRITE(6,*) 'YOU MUST CHOOSE UPPER CASE "Y" OR "N".
      +SELECT AGAIN.'
               GOTO 520
          ENDIF
C
510    CALL EXCMS ('CLRSCRN')
C
       WRITE (41,700) SMODE
       WRITE (41,701) MODE
       WRITE (41,712) EMODE
       WRITE (41,702) NODE
       WRITE (41,703) RM
       WRITE (41,704) SAMPT
       WRITE (41,705) DAMP
       WRITE (41,713) MIN
       WRITE (6,1008)
       WRITE (6,*) '                              PROGRAM RUNNING'
C
C      ********        NOISE AXIS INPUT AND LOCATION        **********
C
       IF(AXIS.EQ.1)THEN
          IMPX = 1.0D0/SAMPT
       ELSEIF(AXIS.EQ.2)THEN
          IMPY = 1.0D0/SAMPT
       ELSEIF(AXIS.EQ.3)THEN
          IMPZ = 1.0D0/SAMPT
       ELSEIF(AXIS.EQ.4)THEN
          IMPX = 1.0D0/SAMPT
          IMPY = 1.0D0/SAMPT
          IMPZ = 1.0D0/SAMPT
       ENDIF
C
```

```
      COUNT = 0
C
C     ************      INITIALIZE MATRICES      ****************
C
      DO 40 I = 1,188
         DO 45 J = 1,188
            PHI(I,J) = 0.0
45       CONTINUE
40    CONTINUE
C
      DO 60 I = 1,188
         DO 65 J = 1,3
            B(I,J) = 0.0
            BN(I,J) = 0.0
65       CONTINUE
60    CONTINUE
C
      DO 70 K = 7,100
         X1(K) = 0.0
         X2(K) = 0.0
         MODEN(K) = 0.0
         RMODEN(K) = 0.0
70    CONTINUE
C
      DO 75 I = 1,100
         READ(42,1040) (UG69(I,K),K=1,3)
         READ(43,1040) (UG23(I,K),K=1,3)
         READ(44,1040) (UG55(I,K),K=1,3)
75    CONTINUE
C
C     ***********************************************************
C     *****              BEGIN MAIN PROGRAM                 *****
C     *****   ESTABLISH PHII, PHI, B AND BN MATRICES        *****
C     ***********************************************************
C
      DO 600 I = SMODE,MODAL
         WN = DBLE(SQRT(LAMA(I)))
         GMA = DAMP*WN/2.0
         EGT = DEXP(-GMA*SAMPT)
         W1 = DSQRT((WN**2)-(GMA**2))
         COSW1T = DCOS(W1*SAMPT)
         SINW1T = DSIN(W1*SAMPT)
C
         IF(WN.EQ.0)THEN
              PHII(1,1,I) = EGT*COSW1T
              PHII(1,2,I) = SAMPT
              PHII(2,1,I) = 0
              PHII(2,2,I) = EGT*COSW1T
C
              GAMMA(1,I) = 0
              GAMMA(2,I) = 0
         ELSE
C
         PHII(1,1,I) = EGT*(COSW1T + (GMA*(W1**(-1)))*SINW1T)
         PHII(1,2,I) = (W1**(-1))*EGT*SINW1T
         PHII(2,1,I) = -(WN**2)*(W1**(-1))*EGT*SINW1T
```

```
           PHII(2,2,I) = EGT*(COSW1T - (GMA*(W1**(-1)))*SINW1T)
C
           GAMMA(1,I) = (WN**(-2))*(1.0D0-EGT*(COSW1T+(GMA/W1)*SINW1T))
           GAMMA(2,I) = (W1**(-1))*EGT*SINW1T
C
           ENDIF
C
600    CONTINUE
C
       V = 1
C
       DO 610 K = SMODE,MODAL
C
           PHI(V,V) = PHII(1,1,K)
           PHI(V,V+1) = PHII(1,2,K)
           PHI(V+1,V) = PHII(2,1,K)
           PHI(V+1,V+1) = PHII(2,2,K)
C
           B(V,1) = GAMMA(1,K)*DBLE(UG69(K,1))
           B(V,2) = GAMMA(1,K)*DBLE(UG69(K,2))
           B(V,3) = GAMMA(1,K)*DBLE(UG69(K,3))
           B(V+1,1) = GAMMA(2,K)*DBLE(UG69(K,1))
           B(V+1,2) = GAMMA(2,K)*DBLE(UG69(K,2))
           B(V+1,3) = GAMMA(2,K)*DBLE(UG69(K,3))
C
           V = V+2
C
610    CONTINUE
C
           DO 605 I = 1,100
              UGVEX(I,1) = 0.0
              UGVEX(I,2) = 0.0
              UGVEX(I,3) = 0.0
                IF(NODE.EQ.23) THEN
                   UGVEX(I,1) = UG23(I,1)
                   UGVEX(I,2) = UG23(I,2)
                   UGVEX(I,3) = UG23(I,3)
                ELSEIF(NODE.EQ.55) THEN
                   UGVEX(I,1) = UG55(I,1)
                   UGVEX(I,2) = UG55(I,2)
                   UGVEX(I,3) = UG55(I,3)
                ELSEIF(NODE.EQ.69) THEN
                   UGVEX(I,1) = UG69(I,1)
                   UGVEX(I,2) = UG69(I,2)
                   UGVEX(I,3) = UG69(I,3)
                ENDIF
605        CONTINUE
C
       V = 1
C
       DO 620 K = SMODE,MODAL
C
           BN(V,1) = GAMMA(1,K)*DBLE(UGVEX(K,1))
           BN(V,2) = GAMMA(1,K)*DBLE(UGVEX(K,2))
           BN(V,3) = GAMMA(1,K)*DBLE(UGVEX(K,3))
           BN(V+1,1) = GAMMA(2,K)*DBLE(UGVEX(K,1))
```

```
      BN(V+1,2) = GAMMA(2,K)*DBLE(UGVEX(K,2))
      BN(V+1,3) = GAMMA(2,K)*DBLE(UGVEX(K,3))
C
      V = V+2
C
620   CONTINUE
C
550   CONTINUE
C
C     ***********   ESTABLISH H, F AND R MATRICES    ***********
C
      DO 50 I = 1,NH
         DO 55 J = 1,NH
            H(I,J) = 0.0
            F(I,J) = 0.0
            G(I,J) = 0.0
            IF(I.LE.3)THEN
              L(I,J) = 0.0
            ENDIF
55       CONTINUE
50    CONTINUE
C
      DO 61 I = 1,3
         DO 66 J = 1,3
               R(I,J) = 0.0
               RINV(I,J) = 0.0
66       CONTINUE
61    CONTINUE
C
      KQ = 1
      DO 80 K = SMODE,EMODE
         H(KQ,KQ) = DBLE(LAMA(K))
         H(KQ+1,KQ+1) = 1.0D0
         KQ = KQ+2
80    CONTINUE
C
      K = 0
      DO 85 K = 1,3
         R(K,K) = RM
         RINV(K,K) = 1.0D0/RM
85    CONTINUE
C
      DO 88 I = 1,2*MODE
         DO 89 J = 1,2*MODE
            F(I,J) = PHI(I,J)
89       CONTINUE
88    CONTINUE
C
C     *******        PREPARE G MATRIX FOR RICDSD       ************
C
         CALL MATRAN(B,188,2*MODE,3,BT,3)
C
         CALL MATMUL(B,188,2*MODE,3,RINV,3,3,TEMP,NH)
C
         CALL MATMUL(TEMP,NH,2*MODE,3,BT,3,2*MODE,G,NG)
C
```

```
C
C     ***********************************************************
C                    BEGIN RICCATI GAIN CALCULATIONS
C     ***********************************************************
C
      CALL RICDSD(NF,NG,NH,NZ,2*MODE,4*MODE,F,G,H,Z,W,ER,EI,WORK,
     +            SCALE,ITYPE,IPVS)
C
         WRITE (6,*) ' '
         WRITE(41,*) ' '
      WRITE (41,110)
110   FORMAT (/' THE RICCATI SOLUTION IS:'/)
      DO 120 I = 1,2*MODE
         WRITE (41,1050) (H(I,J),J=1,2*MODE)
         WRITE(41,*) ' '
120   CONTINUE
      WRITE (6,130)
      WRITE (41,130)
130   FORMAT (/' THE CLOSED LOOP EIGENVALUES ARE:'/)
      DO 140 I = 1,2*MODE
         WRITE (6,*) ER(I),EI(I)
         WRITE (41,*) ER(I),EI(I)
140   CONTINUE
      WRITE (6,150) WORK(1)
      WRITE (41,150) WORK(1)
150   FORMAT (/' CONDITION ESTIMATE IS:',D26.18)
C
C     ************      COMPUTE GAIN MATRIX  L      *************
C
         CALL MATMUL(BT,3,3,2*MODE,H,NH,2*MODE,TEMP1,3)
C
         CALL MATMUL(TEMP1,3,3,2*MODE,B,188,3,RR,3)
C
         DO 103 I = 1,3
              DO 104 J=1,3
                   RR(I,J)=R(I,J)+RR(I,J)
104           CONTINUE
103      CONTINUE
C
      CALL DLINDS(3,RR,3,RRINV,3)
C
      CALL MATMUL(RRINV,3,3,3,TEMP1,3,2*MODE,BT,3)
C
      CALL MATMUL(BT,3,3,2*MODE,PHI,188,2*MODE,L,3)
C
      WRITE(41,*) ' '
      WRITE(41,*) '  GAIN MATRIX   L '
      WRITE(41,*) ' '
      WRITE(41,*) '      ROW 1                ROW 2                ROW 3'
      DO 9155 I=1,2*MODE
              WRITE(41,1040) (L(J,I),J=1,3)
9155  CONTINUE
      WRITE(41,*) ' '
C
C     ***********************************************************
C     *****         COMPUTATION OF TORQUES AND COSTS        *****
```

```
C     ***********************************************************
C
9000  COUNT = 0
      TOTCST = 0.0D0
      TIME = 0.0
C
C     ****************************************************************
C     *****  SETS LOOP FOR THE NUMBER OF ITERATIONS NECESSARY   *****
C     *****  TO OBSERVE THE SYSTEM FOR DESIRED LENGTH OF TIME   *****
C     ****************************************************************
C
      LOOP = INT((MIN*60.0)/SAMPT)
      PRNT = INT(((MIN*60.0)/SAMPT)/100.0)
      PRNTG = INT(((MIN*60.0)/SAMPT)/2000.0)
C
      DO 200 N  = 0, LOOP
         TIME = DBLE(N)*SAMPT
C
         IF(N.EQ.0)THEN
            IMPLSX = IMPX
            IMPLSY = IMPY
            IMPLSZ = IMPZ
         ELSE
            IMPLSX = 0.0D0
            IMPLSY = 0.0D0
            IMPLSZ = 0.0D0
         ENDIF
C
C     ***********************************************************
C     **********       CONTROL TORQUE EQUATIONS       ***********
C     ***********************************************************
C
      SUM1 = 0.0D0
      SUM2 = 0.0D0
      SUM3 = 0.0D0
C
      DO 210 CT = 1, MODE
         CTADJ = CT + (SMODE - 1)
         SUM1 = SUM1 + L(1,2*CT-1)*X1(CTADJ) + L(1,2*CT)*X2(CTADJ)
         SUM2 = SUM2 + L(2,2*CT-1)*X1(CTADJ) + L(2,2*CT)*X2(CTADJ)
         SUM3 = SUM3 + L(3,2*CT-1)*X1(CTADJ) + L(3,2*CT)*X2(CTADJ)
210   CONTINUE
      TCX = SUM1*(-1.0D0)
      TCY = SUM2*(-1.0D0)
      TCZ = SUM3*(-1.0D0)
C
C     ************   UNCONTROLLED SYSTEM TORQUES    ************
C
C     TCX = 0.0D0
C     TCY = 0.0D0
C     TCZ = 0.0D0
C
C     ***********************************************************
C
      IF(N.EQ.0)THEN
         WRITE (32,*) 'IMPULSE X AXIS, IMPULSE Y AXIS, IMPULSE Z AXIS'
```

```
         WRITE (32,*) ' '
         WRITE (32,1040) IMPLSX, IMPLSY, IMPLSZ
         WRITE (32,*) ' '
         WRITE (32,*) 'CONTROL TORQUES TCX, TCY, TCZ'
         WRITE (32,*) ' '
      ENDIF
C
         IF (N.LE.20) THEN
             WRITE (32,2000) TIME, TCX, TCY, TCZ
         ENDIF
C
         IF (MOD(N,PRNTG).EQ.0) THEN
C          WRITE(30,1036) TIME,X1(7),X1(10),X1(30),X1(50),X1(80),X1(100)
           WRITE(30,1036) TIME,X1(7),X1(10),X1(20),X1(30),X1(40),X1(50)
         ENDIF
C
C     ************************************************************
C     ********     SYSTEM COST FUNCTION CALCULATION      ********
C     ************************************************************
C
      SUMC = 0.0D0
      ENERGY = 0.0D0
      CNTCST = 0.0D0
      COST = 0.0D0
C
      DO 230 CF = 7,MODAL
         MODEN(CF) = MODEN(CF)+(X1(CF)**2)*LAMA(CF)+X2(CF)**2
         SUMC = SUMC+(X1(CF)**2)*LAMA(CF)+X2(CF)**2
230   CONTINUE
C
      ENERGY = SUMC
      CNTCST = (TCX**2)*RM+(TCY**2)*RM+(TCZ**2)*RM
      COST = ENERGY + CNTCST
      TOTCST = TOTCST + COST
C
         IF (MOD(N,PRNT).EQ.0) THEN
            COUNT = COUNT+1
            WRITE(33,2000) TIME,ENERGY,CNTCST,COST
         ENDIF
C
C     ************************************************************
C     ********          STATE UPDATE EQUATIONS          **********
C     ************************************************************
C
         DO 220 KA = 7,MODAL
         K = KA-6
C
         X1T=PHII(1,1,KA)*X1(KA)+PHII(1,2,KA)*X2(KA)+B((2*K-1),1)*TCX+
     +       B((2*K-1),2)*TCY+B((2*K-1),3)*TCZ+BN((2*K-1),1)*IMPLSX+
     +       BN((2*K-1),2)*IMPLSY+BN((2*K-1),3)*IMPLSZ
C
         X2T=PHII(2,1,KA)*X1(KA)+PHII(2,2,KA)*X2(KA)+B(2*K,1)*TCX+
     +       B(2*K,2)*TCY+B(2*K,3)*TCZ+BN(2*K,1)*IMPLSX+BN(2*K,2)*
     +       IMPLSY+BN(2*K,3)*IMPLSZ
C
         X1(KA) = X1T
```

```
      X2(KA) = X2T
220   CONTINUE
C
200  CONTINUE
C
     WRITE (35,3000) COUNT
     RTOTAL = TOTCST
     WRITE (34,3002) MODE,RTOTAL
C
     DO 235 K = 7,MODAL
        RMODEN(K) = MODEN(K)
        WRITE (31,3002) K, RMODEN(K)
235  CONTINUE
C
666  CONTINUE
C
C    ***********************************************************
C    *****    CHANGE THE VALUE OF R FOR NEXT RUN???        *****
C    ***********************************************************
C
     WRITE (6,1008)
540  WRITE (6,*) 'DO YOU WANT A NEW  R  VALUE?   (Y/N)    *CAPS ONLY '
     READ(*,1010) RAGAIN
        IF(RAGAIN.EQ.'Y') THEN
            WRITE (6,*) 'ENTER NEW  R  VALUE:      '
            READ (6,*) RM
            GOTO 550
        ELSEIF(RAGAIN.EQ.'N') THEN
            GOTO 530
        ELSE
            WRITE(6,*) 'YOU MUST CHOOSE "Y" OR "N", SELECT AGAIN.'
            GOTO 540
        ENDIF
C
C    ***********************************************************
C    *****            RUN PROGRAM AGAIN OR QUIT???         *****
C    ***********************************************************
C
530  WRITE(6,*) 'DO YOU WANT ANOTHER RUN?     (Y/N)       *CAPS ONLY '
     READ(*,1010) AGAIN
        IF(AGAIN.EQ.'Y') THEN
            GOTO 500
        ELSEIF(AGAIN.EQ.'N') THEN
            GOTO 599
        ELSE
            WRITE(6,*) 'YOU MUST CHOOSE "Y" OR "N", SELECT AGAIN.'
            GOTO 530
        ENDIF
C
C    ***********************************************************
C    *****                FORMAT STATEMENTS                *****
C    ***********************************************************
C
700  FORMAT (' ','STARTING MODE NUMBER:   ',I2)
701  FORMAT (' ','NUMBER OF MODES SCANNED:   ',I2)
702  FORMAT (' ','NOISE INPUT NODE:   ',I3)
```

```
703   FORMAT (' ','INITIAL R VALUE:   ',E12.4)
704   FORMAT (' ','SAMPLING TIME:   ',E12.4)
705   FORMAT (' ','DAMPING FACTOR:   ',E12.4)
712   FORMAT (' ','LAST CONTROLLED MODE:   ',I2)
713   FORMAT (' ','OBSERVATION TIME:   ',F5.1,' MINUTES')
1001  FORMAT(1X,A6)
1002  FORMAT(1X,8E15.8)
1004  FORMAT(1X,//)
1005  FORMAT(1X,60X,E11.5)
1008  FORMAT(1X,////)
1010  FORMAT(A1)
1035  FORMAT(' ',F7.2,2X,5(E12.6,2X))
1036  FORMAT(' ',F7.2,1X,6(E11.5,1X))
1040  FORMAT(' ',3(E15.8,5X))
1050  FORMAT(' ',4(E12.6,2X))
2000  FORMAT(1X,F7.2,3X,3(E15.8,3X))
2001  FORMAT(' ',T5,E15.8)
3000  FORMAT(I4)
3001  FORMAT(F7.2,2X,E12.5)
3002  FORMAT(I3,2X,E12.5)
C
599   STOP
      END
C
C     ************************************************************
C                  SUBROUTINE TO MATRIX MULTIPLY
C     ************************************************************
C
      SUBROUTINE MATMUL(M1,LD1,R1,C1,M2,LD2,C2,MP,LD3)
         INTEGER R1,C1,C2,LD1,LD2,LD3
         REAL*8 M1(LD1,1),M2(LD2,1),MP(LD3,1),SUM
C
         DO 650 I = 1,R1
              DO 660 J = 1,C2
                   SUM = 0.0D0
                 DO 670 K = 1,C1
                   SUM = SUM+M1(I,K)*M2(K,J)
670              CONTINUE
               MP(I,J) = SUM
660           CONTINUE
650      CONTINUE
      RETURN
      END
C
C     ************************************************************
C                 SUBROUTINE TO TRANSPOSE A MATRIX
C     ************************************************************
C
      SUBROUTINE MATRAN(MX,LDX,R1,C1,MT,LDT)
         INTEGER R1,C1,I,J,LDX,LDT
         REAL*8 MX(LDX,1),MT(LDT,1)
C
         DO 680 I = 1,R1
              DO 690 J = 1,C1
```

```
                 MT(J,I) = MX(I,J)
690          CONTINUE
680       CONTINUE
      RETURN
      END
```

APPENDIX B. MODAL DATA

A. MODAL SLOPES FOR NODE 69

The modal slopes for the control node are:

Mode	x'_{69_x}	x'_{69_y}	x'_{69_z}
1	-.142743E-15	0.710767E-14	0.586281E-15
2	-.148348E-14	-.127912E-15	0.255942E-14
3	-.969692E-15	-.174564E-13	-.397837E-15
4	0.203930E-04	-.264468E-14	0.151293E-15
5	0.138406E-06	0.311234E-04	-.300749E-15
6	0.178740E-06	0.127699E-06	0.254676E-04
7	-.134835E-04	0.838629E-06	0.142569E-05
8	-.153043E-05	-.239363E-05	-.112018E-04
9	0.814600E-07	-.578348E-05	0.222256E-06
10	0.353555E-06	-.310832E-05	0.758450E-06
11	0.131939E-05	-.101384E-06	-.389093E-07
12	0.446746E-08	0.544769E-08	-.185488E-07
13	-.637278E-07	-.771111E-07	-.134823E-06
14	0.485695E-07	0.424607E-06	0.352257E-05
15	0.820930E-05	-.266580E-06	-.907079E-06
16	-.164529E-05	-.160255E-05	-.814523E-05
17	-.282375E-06	-.300355E-05	-.400149E-06
18	-.813852E-06	0.198735E-04	-.218691E-05
19	0.378965E-07	0.102157E-04	0.725769E-05
20	0.932392E-06	-.303975E-04	0.161449E-04
21	-.267182E-05	0.312356E-04	-.179908E-05
22	0.229879E-06	0.223818E-04	-.319076E-05
23	0.285227E-05	-.181574E-04	-.719100E-05
24	0.412678E-06	0.988285E-05	-.903574E-05
25	-.644907E-05	0.360769E-04	0.150066E-04
26	-.189005E-04	-.493369E-05	-.250412E-05
27	-.459131E-05	-.619046E-05	-.950471E-05
28	-.225840E-04	-.221983E-05	0.592307E-05
29	0.103822E-05	-.188184E-04	0.313188E-04
30	-.263199E-04	-.805009E-04	0.148650E-04
31	0.648130E-04	-.282148E-04	-.428264E-05
32	-.228027E-06	0.228963E-04	0.633264E-05
33	-.112004E-04	0.246497E-05	0.613080E-05
34	0.102071E-04	-.431311E-05	-.611008E-04
35	-.270660E-04	-.242697E-05	-.268007E-04
36	0.460758E-04	0.904509E-05	0.504665E-04
37	-.902668E-05	0.498229E-05	-.117346E-04
38	0.858209E-05	0.431123E-04	0.195920E-04
39	0.526097E-06	0.650709E-05	0.118511E-04
40	-.574919E-05	-.238998E-05	0.512316E-05
41	0.202286E-04	-.258154E-05	0.525319E-06
42	-.434762E-05	0.560747E-05	0.136490E-05
43	-.630231E-05	0.534208E-06	0.843876E-05
44	0.176625E-04	0.361249E-04	-.134481E-05

45	0.569169E-05	0.551343E-04	-.201197E-04
46	-.117018E-04	0.223734E-04	0.477889E-04
47	0.461803E-05	-.377577E-04	-.261882E-04
48	-.433001E-04	-.440331E-05	-.113462E-04
49	-.464888E-05	-.449081E-04	0.412939E-05
50	-.414528E-04	-.178550E-04	0.141417E-04
51	0.184853E-03	0.113767E-03	0.271882E-04
52	0.854272E-04	-.281916E-03	-.348608E-04
53	-.202251E-04	-.143891E-04	0.226105E-04
54	-.198538E-03	-.786720E-04	0.135269E-03
55	-.346776E-04	-.355351E-04	0.915379E-04
56	-.231472E-04	-.679132E-04	-.230515E-03
57	-.104904E-03	0.879668E-04	-.157236E-03
58	-.801624E-04	-.121128E-04	-.450519E-04
59	-.175262E-06	-.250548E-04	0.727408E-04
60	-.576008E-05	0.237761E-05	-.260183E-04
61	-.201878E-04	0.397783E-04	-.283535E-04
62	-.863867E-04	0.899741E-04	-.602400E-04
63	-.191947E-04	0.826670E-05	-.190661E-04
64	0.680648E-05	0.390045E-04	-.166463E-04
65	0.879968E-04	-.866666E-04	0.418776E-04
66	0.340529E-04	-.381914E-04	0.594181E-05
67	0.436815E-04	-.564851E-04	0.610520E-04
68	0.414499E-05	0.677411E-05	-.348511E-04
69	0.113333E-04	0.361770E-05	-.437398E-05
70	0.683881E-05	0.380880E-04	0.927638E-04
71	-.209233E-04	-.101020E-03	-.173409E-03
72	0.139212E-04	-.351058E-04	-.216052E-04
73	-.781000E-05	0.956189E-04	0.624928E-04
74	-.395466E-04	0.122112E-03	-.282780E-04
75	0.390070E-04	-.141707E-03	-.230823E-04
76	-.830148E-04	0.505458E-03	0.651406E-04
77	0.340632E-05	0.147727E-03	0.746988E-05
78	0.277271E-04	0.781471E-03	0.522312E-04
79	0.247201E-04	0.220736E-03	0.189983E-04
80	-.102019E-04	-.521635E-03	-.546194E-04
81	0.630463E-04	0.168479E-04	0.131902E-05
82	0.346486E-04	0.416642E-04	-.914349E-05
83	0.868904E-04	-.907505E-05	-.486855E-05
84	-.906340E-06	0.259945E-04	-.648830E-04
85	0.756324E-05	-.201949E-04	0.626768E-06
86	-.627706E-04	0.310321E-04	0.137257E-04
87	-.404970E-04	0.743951E-04	-.414458E-04
88	-.921087E-05	0.196669E-04	-.117281E-04
89	-.117445E-03	0.407227E-04	0.422231E-05
90	0.213971E-04	-.175180E-04	-.251707E-04
91	-.211740E-04	0.355053E-04	-.727892E-04
92	0.351910E-04	-.319706E-04	-.254178E-04
93	-.168226E-04	0.690158E-05	0.474687E-04
94	0.602851E-04	-.458706E-04	0.186557E-04
95	-.481965E-06	-.425322E-05	-.215564E-04
96	-.173202E-04	0.708430E-05	0.677372E-04
97	0.174004E-04	0.375807E-05	-.884054E-04
98	0.162886E-04	-.303175E-04	0.273712E-04
99	0.450802E-05	-.338604E-04	0.267023E-04
100	-.866571E-05	0.443611E-04	-.185604E-04

B. MODAL SLOPES FOR NODE 23

The modal slopes for the shuttle docking point are:

Mode	X'_{23_x}	X'_{23_y}	X'_{23_z}
1	0.673765E-16	0.838118E-14	0.323424E-15
2	-.147542E-14	0.821245E-16	0.224082E-14
3	-.563509E-15	-.177485E-13	-.293237E-15
4	0.203930E-04	-.250955E-14	0.137130E-15
5	0.138406E-06	0.311234E-04	-.179575E-15
6	0.178740E-06	0.127699E-06	0.254676E-04
7	-.297725E-05	0.259121E-05	0.188995E-06
8	0.108426E-06	-.218289E-05	-.757105E-06
9	0.874741E-06	-.520435E-06	0.310332E-07
10	0.544216E-07	0.489004E-06	0.271757E-07
11	0.423941E-05	-.304773E-07	-.920528E-07
12	-.224354E-08	0.106262E-07	0.663429E-08
13	0.529625E-09	-.345344E-07	-.281970E-06
14	-.199214E-07	0.366805E-06	0.381323E-05
15	0.189359E-05	-.125987E-05	-.133739E-06
16	-.596798E-07	-.163349E-05	-.575339E-06
17	0.142601E-04	-.108203E-05	-.122127E-05
18	-.384021E-06	0.156167E-04	0.679930E-05
19	-.324575E-05	0.235817E-05	0.138343E-04
20	0.326795E-05	-.151153E-04	0.304402E-04
21	0.375382E-06	0.217470E-04	-.124316E-05
22	-.605343E-05	0.235363E-05	-.495751E-05
23	-.196447E-05	0.926331E-06	-.137887E-04
24	0.127186E-05	0.372716E-04	-.146189E-04
25	-.100225E-04	0.727055E-04	0.207894E-04
26	-.490188E-04	-.157117E-04	-.580300E-05
27	0.356173E-05	0.357437E-04	-.124972E-04
28	-.168406E-04	0.473385E-05	0.346565E-05
29	0.129236E-05	-.170470E-05	0.147334E-04
30	0.928338E-05	0.150484E-05	-.155595E-04
31	-.186883E-04	-.573203E-05	-.105849E-05
32	0.105083E-05	-.898290E-05	0.720034E-05
33	-.775806E-05	0.928297E-05	-.142005E-05
34	-.899212E-05	0.928592E-05	0.143380E-04
35	-.213398E-05	0.211233E-04	0.395609E-05
36	-.125435E-04	-.743126E-05	-.722542E-05
37	0.560519E-05	-.327904E-04	0.171513E-04
38	-.878226E-05	-.202701E-04	-.279081E-04
39	-.753088E-05	-.864820E-05	-.615209E-05
40	-.170216E-04	0.996299E-06	0.816306E-05
41	-.272982E-04	0.793742E-05	0.774993E-05
42	-.339703E-04	0.185176E-04	-.171827E-04
43	-.205737E-04	-.591944E-05	0.824942E-05
44	-.136327E-04	-.310077E-05	-.669059E-05
45	-.331961E-05	0.807570E-05	-.876419E-05
46	-.407345E-05	-.134132E-04	0.204464E-04
47	0.100101E-04	0.483764E-05	-.328345E-05
48	-.465857E-05	0.894650E-05	-.229645E-04
49	0.209734E-04	0.851673E-05	-.123184E-04

50	0.735291E-05	-.111341E-04	0.610389E-05
51	-.186840E-04	-.141596E-05	-.171407E-04
52	-.305064E-05	0.291705E-05	0.291798E-05
53	0.612794E-05	-.259560E-05	-.301641E-05
54	0.224150E-04	0.356715E-05	-.246719E-04
55	0.104824E-04	0.472466E-05	-.130989E-04
56	0.365390E-04	0.156131E-04	0.137881E-04
57	-.252152E-04	-.667200E-05	0.238919E-04
58	0.466116E-05	0.362378E-05	0.206094E-05
59	0.512889E-04	0.156742E-04	0.736908E-05
60	-.886299E-05	-.173993E-05	0.145529E-04
61	-.118827E-04	-.638678E-05	-.341865E-05
62	-.116328E-04	-.127572E-04	-.118802E-04
63	-.334758E-04	-.460213E-05	0.951309E-05
64	0.282063E-04	-.181336E-04	-.324784E-05
65	0.298193E-04	0.928493E-05	0.362846E-05
66	-.779794E-05	-.372176E-05	0.472953E-05
67	-.799965E-05	0.153350E-04	0.151446E-04
68	0.598300E-05	-.136686E-04	-.107572E-04
69	0.445989E-05	0.451041E-05	-.106900E-04
70	0.125375E-04	0.326667E-04	0.261328E-04
71	0.285552E-04	-.153862E-04	-.654984E-04
72	0.120996E-04	0.814614E-05	-.135752E-04
73	0.163203E-04	-.717707E-05	0.268940E-04
74	0.494562E-05	-.799071E-05	-.108450E-04
75	-.106240E-05	0.111795E-04	-.123315E-04
76	0.146456E-04	-.175669E-04	0.236911E-04
77	-.113865E-04	0.129361E-04	-.794027E-05
78	-.732011E-05	0.221964E-04	-.244142E-04
79	0.344824E-04	-.552859E-04	0.114108E-04
80	0.136796E-04	-.428437E-05	-.271471E-05
81	0.446742E-04	0.209841E-04	-.299774E-04
82	0.279713E-04	0.522715E-05	-.214934E-04
83	0.875577E-04	-.538057E-04	-.301668E-04
84	-.184704E-04	-.864343E-06	-.676264E-04
85	-.139697E-03	0.337797E-04	-.136488E-04
86	0.458667E-04	-.305818E-04	0.116631E-04
87	0.973355E-04	0.188418E-04	-.690089E-04
88	0.831473E-05	0.494336E-05	-.341722E-04
89	-.262133E-05	0.273551E-04	0.438873E-04
90	0.227642E-03	-.872639E-05	0.445858E-04
91	0.117255E-03	-.306443E-04	-.600956E-04
92	0.704937E-04	0.145022E-03	-.152194E-04
93	0.474469E-04	0.343841E-05	-.256578E-03
94	0.547411E-05	0.452852E-04	-.251172E-04
95	-.209031E-04	0.884218E-05	0.350835E-04
96	0.644381E-04	0.679948E-05	-.126796E-03
97	-.697126E-04	0.470702E-05	0.145893E-03
98	0.332597E-05	-.687407E-04	-.138210E-04
99	0.199743E-04	-.808830E-04	-.320429E-04
100	0.273875E-06	0.755504E-04	0.215716E-04

C. MODAL SLOPES FOR NODE 55

The modal slopes for the alpha-joint are:

Mode	x'_{55_x}	x'_{55_y}	x'_{55_z}
1	0.383728E-15	0.648503E-14	0.259748E-15
2	-.184671E-14	0.488119E-15	0.254360E-14
3	-.128428E-14	-.173839E-13	-.282479E-15
4	0.203930E-04	-.206593E-14	0.167872E-15
5	0.138406E-06	0.311234E-04	-.136062E-15
6	0.178740E-06	0.127699E-06	0.254676E-04
7	0.347220E-04	-.188394E-05	-.411696E-05
8	0.376972E-05	-.259439E-05	0.306946E-04
9	-.431695E-06	0.174547E-04	-.446992E-06
10	-.920262E-06	-.172377E-04	-.173126E-05
11	-.535435E-05	-.177299E-06	0.532138E-07
12	-.147302E-08	0.394847E-08	-.300984E-07
13	-.110737E-06	0.151722E-07	0.147360E-06
14	0.115360E-07	0.168239E-06	0.365397E-05
15	-.203315E-04	0.745574E-06	0.271228E-05
16	0.390441E-05	-.115324E-05	0.235683E-04
17	-.292789E-04	0.345807E-05	0.615796E-06
18	0.113699E-06	0.118908E-04	-.780718E-06
19	-.277056E-05	-.186529E-04	-.182640E-04
20	-.152522E-05	0.248623E-04	-.221035E-04
21	0.593853E-05	0.123425E-03	0.309185E-05
22	-.189305E-05	-.546579E-04	0.235529E-05
23	0.384284E-05	0.763794E-04	0.729427E-05
24	0.406456E-06	-.216016E-04	0.703853E-05
25	0.257490E-05	0.360081E-05	-.223156E-06
26	0.721442E-05	0.133132E-05	0.177993E-05
27	0.260727E-05	0.126390E-04	0.182895E-06
28	0.462707E-04	-.146602E-04	-.139853E-04
29	-.614046E-05	-.740727E-04	-.455291E-04
30	-.592501E-05	0.691365E-04	0.103959E-04
31	0.179054E-04	0.264025E-04	-.505956E-05
32	0.405053E-05	0.171347E-03	-.449067E-04
33	-.224200E-04	0.513369E-05	0.207218E-05
34	-.843328E-06	-.381094E-05	-.136130E-04
35	-.369632E-04	0.400812E-05	-.298823E-05
36	-.119381E-04	0.402865E-05	0.708221E-05
37	0.332582E-05	0.467050E-05	0.561253E-05
38	0.822083E-05	-.498378E-05	0.467743E-05
39	-.367060E-04	-.183247E-05	0.124242E-04
40	0.145835E-03	-.246676E-05	-.855994E-04
41	-.286373E-04	-.133249E-04	-.347179E-04
42	0.225691E-04	0.180725E-04	0.163816E-03
43	-.139748E-03	-.315890E-04	-.898054E-04
44	0.519186E-04	0.885869E-05	0.414642E-05
45	-.247154E-04	0.412903E-05	-.609737E-04
46	-.162735E-05	0.488829E-04	0.506333E-04
47	-.362745E-06	0.808802E-04	-.127292E-04
48	-.227231E-04	0.132977E-04	-.933161E-04
49	0.170334E-05	0.927167E-04	-.673497E-04

50	-.114981E-04	0.473668E-03	0.486152E-04
51	-.623501E-05	0.104797E-03	-.108402E-04
52	-.213905E-05	0.150511E-04	0.308070E-05
53	0.272268E-04	-.967355E-03	0.200135E-04
54	0.113440E-05	0.970653E-04	-.764966E-05
55	0.221595E-05	-.551031E-05	-.170689E-05
56	0.139864E-05	-.428632E-05	0.499335E-05
57	0.405844E-06	-.772733E-05	-.672783E-05
58	0.315819E-05	0.268137E-05	0.702179E-05
59	0.112618E-06	-.469574E-05	-.719491E-05
60	0.915609E-05	0.243004E-04	0.988749E-04
61	0.359368E-04	0.101685E-04	-.146203E-05
62	-.998475E-05	0.299313E-05	0.238850E-05
63	0.112230E-04	0.790803E-05	-.103956E-04
64	-.123858E-04	0.976885E-05	-.362704E-05
65	0.254959E-04	0.762204E-05	0.371769E-05
66	-.826521E-04	-.368223E-04	-.351764E-05
67	0.302862E-04	0.984704E-05	0.626748E-05
68	0.402500E-04	0.276257E-04	-.649850E-05
69	-.346891E-04	-.113285E-05	-.138124E-04
70	0.740900E-05	0.412078E-05	-.168142E-05
71	0.211703E-04	-.141406E-04	0.648243E-05
72	0.967122E-05	-.918167E-04	0.205426E-04
73	0.343108E-04	-.929812E-04	0.276350E-04
74	0.806898E-06	-.339339E-04	0.104327E-04
75	-.339695E-05	0.411311E-05	-.138645E-05
76	0.111561E-05	0.255038E-04	-.528544E-05
77	-.638271E-06	0.133937E-02	0.318582E-04
78	-.770564E-05	-.326199E-03	-.891922E-05
79	0.850119E-05	0.150388E-03	0.703879E-05
80	-.445043E-05	-.410070E-04	-.571701E-05
81	0.113578E-04	0.326288E-04	0.955806E-05
82	-.453730E-05	-.881092E-04	-.358309E-04
83	-.105073E-05	0.231661E-04	0.407640E-05
84	0.574304E-06	-.905933E-05	0.457371E-06
85	0.563069E-06	-.166207E-04	-.180061E-05
86	0.156726E-05	0.601892E-05	-.789847E-06
87	-.379922E-05	-.655688E-05	-.268607E-05
88	0.132049E-05	0.148011E-04	0.312786E-05
89	-.780189E-05	-.385735E-05	-.177576E-05
90	0.637208E-06	0.133292E-04	-.288901E-06
91	-.216137E-05	0.320254E-05	-.771067E-06
92	-.177956E-05	-.211414E-05	-.459527E-05
93	-.234858E-05	-.120165E-04	-.321270E-05
94	-.109741E-04	-.102676E-04	-.305376E-05
95	-.450829E-05	-.861825E-05	0.404654E-06
96	-.516899E-05	-.114223E-04	-.551371E-05
97	-.461506E-05	0.421219E-06	0.286892E-05
98	0.346160E-05	0.566570E-05	0.269516E-05
99	0.381413E-05	0.171670E-04	0.549198E-05
100	-.347057E-05	-.890927E-04	-.448555E-04

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